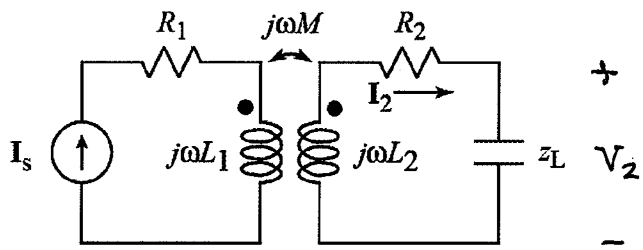
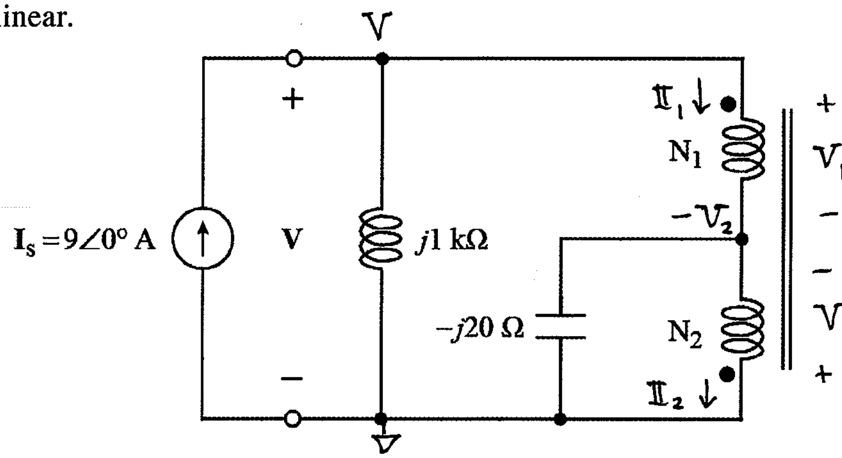
Ex:



$I_s = 2\angle 45°$ A $R_1 = 10\ \Omega$ $j\omega L_1 = j10\ \Omega$ $j\omega M = j15\Omega$

$R_2 = 30\ \Omega$ $j\omega L_2 = j40\Omega$ $z_L = -j10\ \Omega$

a) Calculate the numerical value of phasor current, $\mathbf{I}_2$, flowing into the load impedance, z_L, on the secondary side of the transformer. Note: the transformer is linear.



b) The turns ratio of the transformer is $N_1/N_2 = 6$. Calculate the numerical value of phasor voltage, **V**, shown in the diagram. Note: the transformer is ideal.

sol'n: a) We can use the idea of reflected impedance to find the current, I_1, flowing in the primary and then substitute for I_1 in the voltage equation for the secondary. At least, that is what we could do in general. Here, $I_1 = I_s$, so we can move directly to the voltage equation for the secondary:

$$V_2 = I_1 j\omega M - I_2(j\omega L_2 + R_2) = I_2 z_L$$

or

$$\mathbb{I}_S\, j\omega M = \mathbb{I}_2 (j\omega L_2 + R + Z_L)$$

or

$$\mathbb{I}_2 = \frac{\mathbb{I}_S\, j\omega M}{R_2 + j\omega L_2 + Z_L}$$

or

$$\mathbb{I}_2 = \frac{2\angle 45^\circ \cdot j\,15\,\Omega}{30\,\Omega + j\,40\,\Omega - j\,10\,\Omega}\ \mathrm{A}$$

or

$$\mathbb{I}_2 = 2\angle 45^\circ \frac{j\,15}{30 + j\,30}\ \mathrm{A}$$

or

$$\mathbb{I}_2 = 2\angle 45^\circ \cdot \frac{15\angle 90^\circ}{30\sqrt{2}\angle 45^\circ}\ \mathrm{A}$$

or

$$\mathbb{I}_2 = \frac{1}{\sqrt{2}} \angle 90^\circ\ \mathrm{A} \qquad \text{polar form}$$

$$\mathbb{I}_2 = \frac{1}{2} + j\frac{1}{2}\ \mathrm{A} \qquad \text{rectangular form}$$

b) We use the ideal transformer equations:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{\mathbb{I}_2}{\mathbb{I}_1}$$

And we use Kirchhoff's laws or other circuit-solving techniques. Here, node-voltage is used.

$$\text{node } V: \quad -\mathbb{I}_S + \frac{V}{j\,1\,k\Omega} + \mathbb{I}_1 = 0\ \mathrm{A} \qquad (1)$$

$$\text{node } -V_2: \quad -\mathbb{I}_1 - \frac{V_2}{-j\,20\,\Omega} + \mathbb{I}_2 = 0\,\mathrm{A} \qquad (2)$$

We need one more equation, in addition to the transformer equations. We turn to Kirchhoff's laws and find that we are able to write another equation for $\mathbb{V}$.

$$\mathbb{V} = -\mathbb{V}_2 + \mathbb{V}_1 \qquad (\mathbb{V}\text{-loop}) \ (3)$$

Now we start eliminating variables using the transformer equations first.

$$\mathbb{V}_1 = \mathbb{V}_2 \frac{N_1}{N_2} \quad \text{and} \quad \mathbb{I}_2 = \mathbb{I}_1 \frac{N_1}{N_2}$$

Equation (3) becomes

$$\mathbb{V} = -\mathbb{V}_2 + \mathbb{V}_2 \frac{N_1}{N_2} = \mathbb{V}_2\left(\frac{N_1}{N_2} - 1\right)$$

or

$$\mathbb{V}_2 = \frac{\mathbb{V}}{\frac{N_1}{N_2} - 1} .$$

Equation (2) becomes

$$-\mathbb{I}_1 - \frac{\mathbb{V}}{\left(\frac{N_1}{N_2} - 1\right)(-j20\,\Omega)} + \mathbb{I}_1\left(\frac{N_1}{N_2}\right) = 0\text{ A}$$

or

$$\mathbb{I}_1\left(\frac{N_1}{N_2} - 1\right) = \frac{\mathbb{V}}{\left(\frac{N_1}{N_2} - 1\right)(-j20\,\Omega)}$$

or

$$\mathbb{I}_1 = \frac{\mathbb{V}}{\left(\frac{N_1}{N_2} - 1\right)^2(-j20\,\Omega)} .$$

Equation (1) becomes

$$-\mathbb{I}_S + \frac{V}{j1k\Omega} + \frac{V}{\left(\frac{N_1}{N_2}-1\right)^2(-j20\Omega)} = 0\,A$$

or

$$V\left(\frac{1}{j1k\Omega} + \frac{1}{\left(\frac{N_1}{N_2}-1\right)^2(-j20\Omega)}\right) = \mathbb{I}_S$$

or

$$V = \frac{\mathbb{I}_S}{\frac{1}{j1k\Omega} + \frac{1}{\left(\frac{N_1}{N_2}-1\right)^2(-j20\Omega)}}$$

or

$$V = \frac{9\angle 0^\circ\ A}{\frac{-j}{1k\Omega} + \frac{j}{5^2(+20)\Omega}}$$

or

$$'' = \frac{9\ A}{\frac{-j}{1k\Omega} + \frac{j2}{1k\Omega}}$$

or

$$'' = 9\,A \cdot \frac{1k\Omega}{j}$$

or

$$V = -j9kV = 9kV\angle -90^\circ$$

Note: *We could have solved this more quickly by observing earlier that we really have the following eq'n.*

$$V = \mathbb{I}_S \cdot j1k\Omega \,\Big\|\, \left[\left(\frac{N_1}{N_2}-1\right)^2(-j20\Omega)\right]$$

or

$$V = \mathbb{I}_S \cdot j1k\Omega \,\|\, -j500\Omega = \mathbb{I}_S \cdot j500 \cdot \frac{-2}{1}\,\Omega$$

or

$$V = \mathbb{I}_S \cdot -j1k\Omega = -j9kV$$