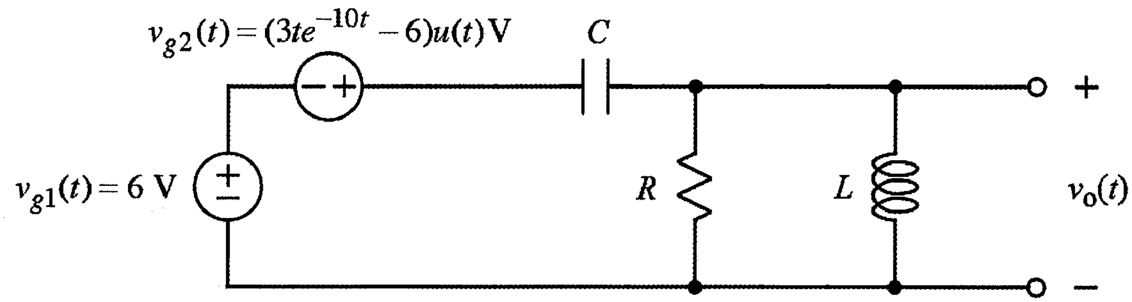
Ex:



Note: The 6 V in the $v_{g1}(t)$ source is always on.

$C = 1\ \mu F$ $\quad R = 100\ \Omega$ $\quad L = 40$ mH

a) Write the Laplace transform, $V_g(s)$, of $v_g(t) = v_{g1}(t) + v_{g2}(t)$.

b) Draw the s-domain equivalent circuit, including source $V_g(s)$, components, initial conditions for C's, and terminals for $V_o(s)$.

c) Write an expression for $V_o(s)$. You may use the parallel operator (for parallel impedances) in your answer.

d) Apply the initial value theorem to find $\lim_{t \to 0^+} v_o(t)$.

SOL'N: a) Note that $\mathcal{L}\{6\} = \mathcal{L}\{6u(t)\} = \frac{6}{s}$.

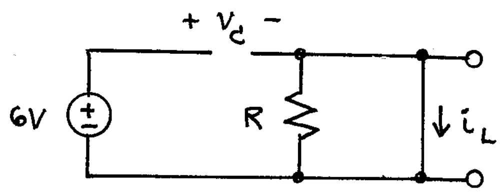
$$\mathcal{L}\{v_{g2}(t)\} = \mathcal{L}\{(3te^{-10t} - 6)u(t)\} \text{ V}$$

$$= \frac{3}{(s+10)^2} - \frac{6}{s} \text{ V}$$

$$V_g(s) = \frac{6}{s} + \frac{3}{(s+10)^2} - \frac{6}{s} \text{ V}$$

$$V_g(s) = \frac{3}{(s+10)^2} \text{ V}$$

b) We find initial conditions at $t=0^-$.
$L =$ wire, $C =$ open, $v_{g1} = 6V$, $v_{g2} = 0V$

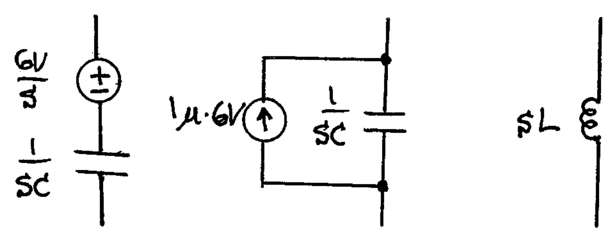


Using an outside v-loop, we find that we must have $v_c = 6V$.

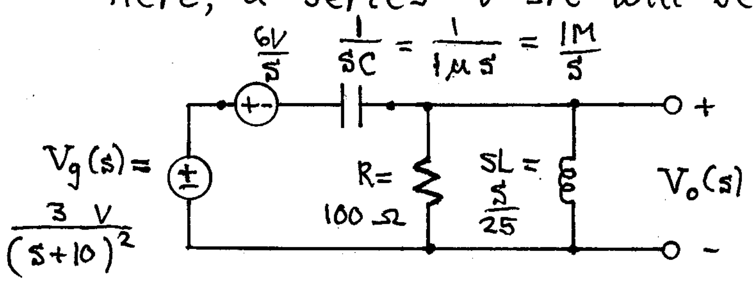
Because we have an open circuit, $i_L = 0A$.

$$v_c(0^-) = 6V, \quad i_L(0^-) = 0A$$

Now we can draw the s-domain circuit. We have the following alternatives for initial conditions on the L and C:



Here, a series v-src will be used for C.



s-domain equivalent circuit

c) We sum voltage sources and use a v-divider.

$$V_o(s) = \left(\frac{3\ V}{(s+10)^2} - \frac{6}{s}V\right)\frac{R\|sL}{R\|sL + 1/sC}$$

d) *Initial value theorem:*

$$\lim_{t\to 0^+} v_o(t) = \lim_{s\to\infty} s\,V(s)$$

$$= \lim_{s\to\infty} s\left[\frac{3V}{(s+10)^2} - \frac{6V}{s}\right]\frac{R\|sL}{R\|sL + \frac{1}{sC}}\ V$$

$$= \lim_{s\to\infty}\left[\frac{3s}{(s+10)^2} - 6\right]\frac{R\|sL}{R\|sL + \frac{1}{sC}}\ V$$

We may ignore all terms in a sum except the highest power of s.

$$= \lim_{s\to\infty}\left[\frac{3s}{s^2} - 6\right]\frac{\frac{RLs}{\cancel{R}+Ls}}{\frac{RLs}{\cancel{R}+Ls}}\ V$$

$$= \lim_{s\to\infty}\left[\underbrace{\frac{3\cancel{s}}{s\cancel{s}}}_{\text{small as } s\to\infty} - 6\right]\underbrace{\frac{\frac{RL\cancel{s}}{L\cancel{s}}}{\frac{\cancel{R}Ls}{Ls}}}_{1}\ V$$

$$= \lim_{s\to\infty} -6\ V$$

$$\lim_{t\to 0^+} v_o(t) = -6\ V$$