Ex:

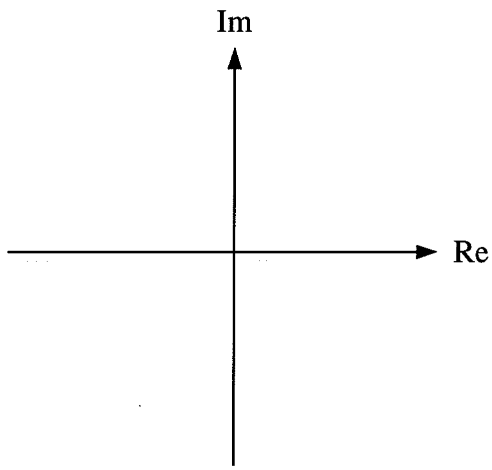
a) Find $\mathcal{L}\{t\,u(t-3)\}$.

b) Find $v(t)$ if $V(s) = \dfrac{s}{s^2 + 24s + 160} + \dfrac{2}{s+1}$.

c) Find $\lim\limits_{t \to 0^+} v(t)$ if $V(s) = \dfrac{4(s^2 + 6s + 25)}{s(s+1)(s+5)}$.

d) Plot and label the values of the poles and zeros of $V(s)$ in the s plane.

$$V(s) = \frac{1}{s+5} + \frac{2}{(s+1)^2 + 9}$$



SOL'N: a) One way to solve this problem is to make $t\,u(t-3)$ look like $v(t-3)\,u(t-3)$. To do so, we write t as $(t-3)+3$:

$$\mathcal{L}\{t\,u(t-3)\} = \mathcal{L}\{[(t-3)+3]\,u(t-3)\}$$

So $v(t-3) = (t-3)+3$ and we use the delay identity:

$$\mathcal{L}\{v(t-a)\,u(t-a)\} = e^{-as}\mathcal{L}\{v(t)\}$$

We have $v(t) = t + 3$; (replace $t-3$ with t).

$$\mathcal{L}\{t+3\} = \frac{1}{s^2} + \frac{3}{s}$$

So $$\mathcal{L}\{t\,u(t-3)\} = e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s}\right)$$

Another approach would be to use the identity for multiplication by t:

$$\mathcal{L}\{t\,v(t)\} = -\frac{d}{ds}\,\mathcal{L}\{v(t)\}$$

So we have $v(t) = u(t-3) = 1\,u(t-3)$, and we use the delay identity with $v(t-3) = 1$, which means $v(t) = 1$. (When we change from $v(t-a)$ to $v(t)$, we shift the function $v(t-a)$ to the left by 'a'. If we have a constant function, shifting it left has no effect — it is still a horizontal line.

So $$\mathcal{L}\{1\,u(t-3)\} = e^{-3s}\,\mathcal{L}\{1\} = \frac{e^{-3s}}{s}$$

Now we use the identity for multiplication by t.

$$\mathcal{L}\{t\,u(t-3)\} = -\frac{d}{ds}\,\frac{e^{-3s}}{s}$$

$$= -(-3)\frac{e^{-3s}}{s} - -\frac{e^{-3s}}{s^2}$$

$$\mathcal{L}\{t\,u(t-3)\} = e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s}\right)$$

b) Use partial fractions, but write the first term as a decaying cos() and sin():

$$V(s) = \frac{s}{s^2 + 24s + 160} + \frac{2}{s+1}$$

$$V(s) = \frac{(s+\overset{a}{12}) - 3(\overset{\omega}{4})}{(s+\underset{a}{12})^2 + \underset{\omega}{4}^2} + \frac{2}{s+1}$$

$$v(t) = \left[e^{-12t}\cos(4t) - 3e^{-12}\sin(4t) + 2e^{-t} \right] u(t)$$

c) Use the initial value theorem:

$$\lim_{t \to 0^+} v(t) = \lim_{s \to \infty} sV(s)$$

$$= \lim_{s \to \infty} \frac{\cancel{s} \cdot 4(s^2 + 6s + 25)}{\cancel{s}(s+1)(s+5)}$$

$$= \lim_{s \to \infty} \frac{4s^2}{s^2}$$

we may ignore additive terms of lower order in s than the highest order term

$$= \lim_{s \to \infty} 4$$

$$\lim_{t \to 0^+} v(t) = 4$$

d) We use a common denominator to write $V(s)$ as a ratio of polynomials in s.

$$V(s) = \frac{1}{s+5} + \frac{2}{(s+1)^2+9}$$

$$= \frac{(s+1)^2 + 9 + 2(s+5)}{(s+5)[(s+1)^2+9]}$$

$$= \frac{s^2 + 2s + 1 + 9 + 2s + 10}{(s+5)[(s+1)^2+9]}$$

$$= \frac{s^2 + 4s + 20}{(s+5)[(s+1)^2+9]}$$

$$V(s) = \frac{(s+2)^2 + 4^2}{(s+5)\ (s+1)^2 + 3^2}$$

zeros are roots of numerator o's
poles " " " denominator x's

zeros: $s = -2 + j4, \quad s = -2 - j4$

poles: $s = -5, \quad s = -1 + j3, \quad s = -1 - j3$