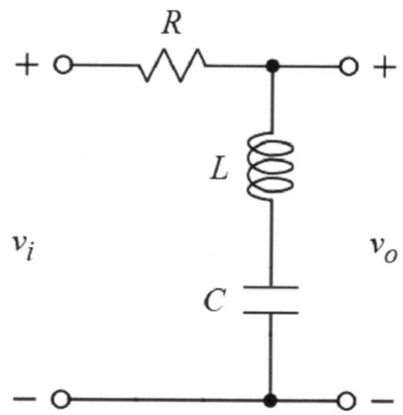
Ex:



$R = 1.5\ \Omega$ $L = 1$ mH $C = \frac{25}{16}$ mF

$$v_i(t) = 2 + \sum_{k=1}^{\infty}\left[\frac{2}{k\pi}\cos(k\omega_0 t) - \frac{1}{k\pi}\sin(k\omega_0 t)\right] \text{V}$$

Write the time-domain expression of the first harmonic (i.e., $k = 1$) of $v_o(t)$.

Note: $\omega_o = 3.2$ k r/s for the Fourier series.

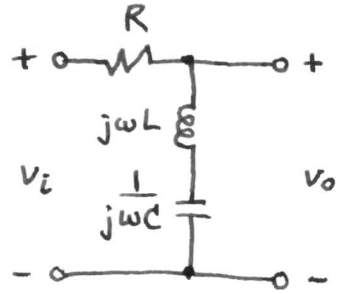
SOL'N: The $k=1$ term of $v_i(t)$ does not include the DC term, $a_v = 2$ V, since the DC term has frequency zero rather than ω_0.

$$v_{i1}(t) = \frac{2}{\pi}\cos(\omega_0 t) - \frac{1}{\pi}\sin(\omega_0 t)$$

Now we convert to a phasor. Note that $P[\cos(\omega_0 t)] = 1$ and $P[\sin(\omega_0 t)] = -j$.

$$V_{i1} = \frac{2}{\pi} - -j\frac{1}{\pi} = \frac{1}{\pi}(2+j)$$

The output phasor for frequency ω_0 may be obtained by a voltage-divider or by multiplying V_{i1} by transfer function $H(j\omega)$ evaluated at $\omega = \omega_0$.



$$H(j\omega) = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{1}{\frac{R}{j\omega L + \frac{1}{j\omega C}} + 1}$$

$$= \frac{1}{1 + \frac{R}{j\left(\omega L - \frac{1}{\omega C}\right)}}$$

$$H(j\omega) = \frac{1}{1 - j\frac{R}{\omega L - \frac{1}{\omega C}}}$$

$$H(j\omega_o) = \frac{1}{1 - j\frac{R}{\omega_o L - \frac{1}{\omega_o C}}}$$

where

$$R = 1.5\,\Omega \qquad \omega_o L = 3.2\ \text{kr/s} \cdot 1\ \text{mH} = 3.2\,\Omega$$

$$\frac{1}{\omega_o C} = \frac{1}{3.2\,\text{kr/s}\ \frac{25}{16}\ \text{mF}} = \frac{1}{5}\,\Omega = 0.2\,\Omega$$

Thus, $\frac{R}{\omega_o L - \frac{1}{\omega_o C}} = \frac{1.5\,\Omega}{3.2\,\Omega - 0.2\,\Omega} = \frac{1.5\,\Omega}{3\,\Omega} = \frac{1}{2}.$

We have $V_{o1} = V_{i1}\ H(j\omega_o)$

$$= \frac{1}{\pi}(2+j)V \frac{1}{1-j\frac{1}{2}}$$

$$= \frac{1}{\pi}(2+j)V \frac{1}{1-j\frac{1}{2}} \frac{1+j\frac{1}{2}}{1+j\frac{1}{2}}$$

$$= \frac{1}{\pi}\frac{(2+j)(1+j\frac{1}{2})}{1+\frac{1}{4}} V$$

$$= \frac{1}{\pi}\left(2-\frac{1}{2}+j+j\right)/\frac{5}{4} V$$

$$= \frac{1}{\pi}\left(\frac{3}{2}+j2\right)\frac{4}{5} V$$

$$V_{o1} = \frac{1}{\pi}\left(\frac{6}{5}+j\frac{8}{5}\right) V$$

$$v_{o1}(t) = \frac{1}{\pi}\left(\frac{6}{5}\right)\cos(\omega_o t) - \frac{1}{\pi}\left(\frac{8}{5}\right)\sin(\omega_o t)\ V$$

or $v_{o1}(t) = 0.382 \cos(3.2\text{k}t)$
$\quad - 0.510 \sin(3.2\text{k}t)\ V$