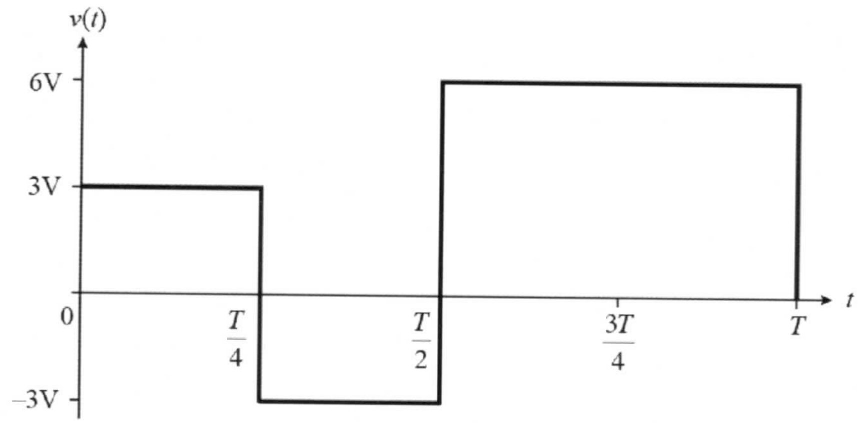
Ex:



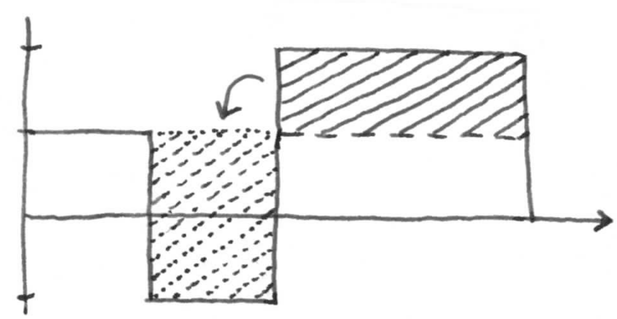
One period, T, of a function $v(t)$ is shown above. The formula for $v(t)$ is

$$v(t)=\begin{cases} 3\text{ V} & 0<t<T/4 \\ \text{-3 V} & T/4<t<T/2 \\ 6\text{ V} & T/2<t<T \end{cases}$$

Find the numerical value of the following coefficients of the Fourier series for $v(t)$:

a) a_v b) a_1 c) b_1 d) b_2

Sol'n: a) a_v is the average height of $v(t)$. We observe that the portion of $v(t)$ above 3V on the right side of $v(t)$ would fill the dip between T/4 and T/2 to a height of 3V. So the average height is $a_v = 3V$.



Using calculus, we find the area under the $v(t)$ function from 0 to T, and we divide by T.

$$a_v = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T/4} 3V\,dt + \int_{T/4}^{T/2} (-3V)\,dt + \int_{T/2}^{T} 6V\,dt\right]$$

$$= \frac{1}{T}\left[3Vt\Big|_0^{T/4} + -3Vt\Big|_{T/4}^{T/2} + 6Vt\Big|_{T/2}^{T}\right]$$

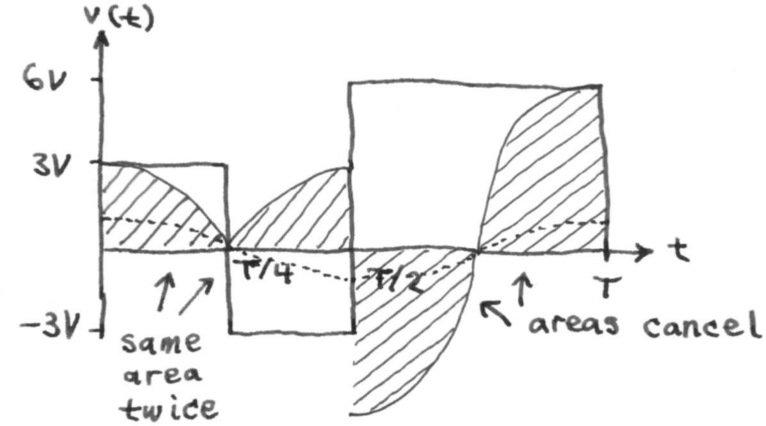
$$= \frac{1}{T}\left[3V\left(\frac{T}{4} - 0\right) - 3V\left(\frac{T}{2} - \frac{T}{4}\right) + 6V\left(T - \frac{T}{2}\right)\right]$$

$$= \frac{1}{T}\left[3V\left(\frac{T}{4}\right) - 3V\left(\frac{T}{4}\right) + 6V\left(\frac{T}{2}\right)\right]$$

$$a_v = \frac{6V}{2} = 3V \quad \checkmark$$

b) $$a_1 = \frac{2}{T}\int_0^T v(t)\cos(\omega_0 t)\,dt$$

Picture:



From the picture, $\int_0^T v(t)\cos(\omega_0 t)\,dt$ is equal to twice the area between 0 and T/4.

$$a_1 = \frac{2(2)}{T}\int_0^{T/4} 3V\cos(\omega_0 t)\,dt$$

$$= \frac{2(2)3V}{T}\left.\frac{\sin(\omega_0 t)}{\omega_0}\right|_0^{T/4}$$

$$= \frac{12V}{T}\left[\frac{\cancelto{1}{\sin\left(\frac{2\pi}{T}\left(\frac{T}{4}\right)\right)} - \cancelto{0}{\sin\left(\frac{2\pi}{T}\cdot 0\right)}}{\frac{2\pi}{T}}\right]$$

$$= \frac{12V}{2\pi}(1)$$

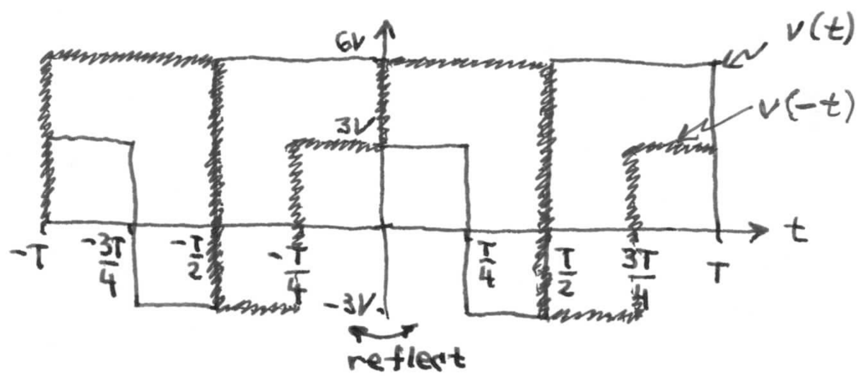
$$a_1 = \frac{6}{\pi}V \quad \text{or} \quad a_1 \approx 1.91\,V$$

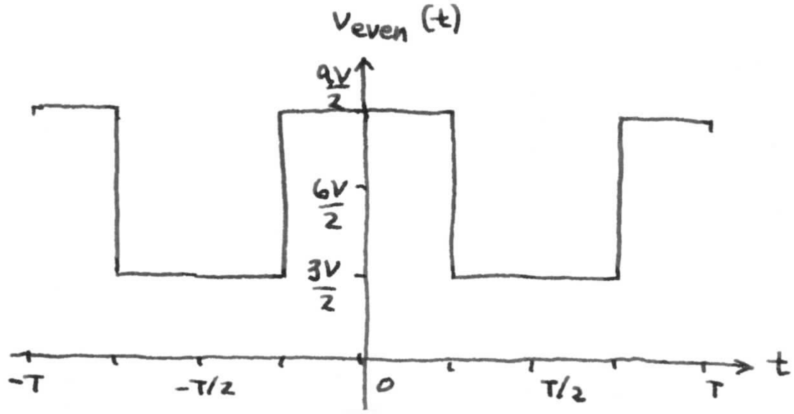
Note: another way to approach this problem is to split $v(t)$ into even and odd parts.

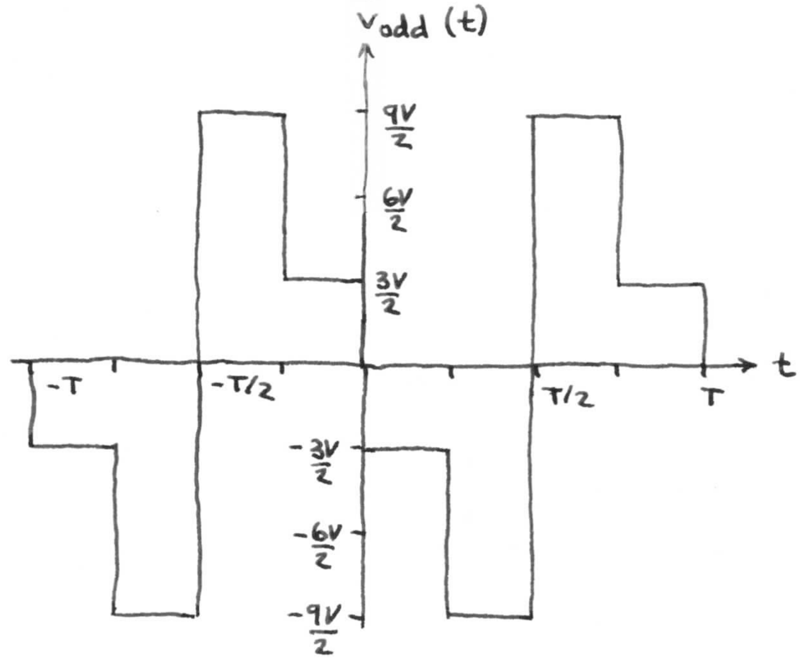
$$v_{even}(t) = \frac{v(t)+v(-t)}{2}$$

$$v_{odd}(t) = \frac{v(t)-v(-t)}{2}$$

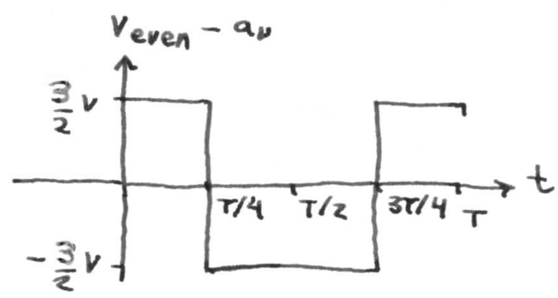
where $v(-t)$ is $v(t)$ reflected around the vertical axis







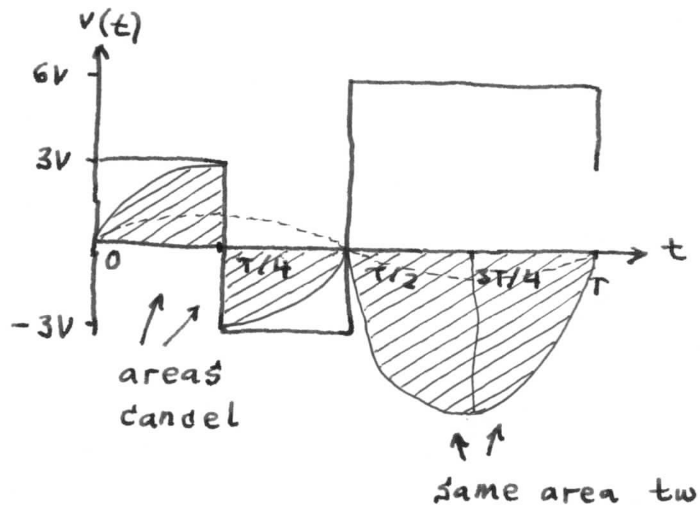
We may also subtract a_0 from $v_{even}(t)$ when calculating coefficients other than a_0.



We would use v_{even} or $v_{even} - a_0$ to calculate a_1. Here, there is little advantage to using v_{even} or v_{odd} instead of the original $v(t)$.

c) $b_1 = \frac{2}{T}\int_0^T v(t)\sin(\omega_0 t)\,dt$

Picture:



From the picture, we may use twice the area between T/2 and 3T/4. If we look at the shapes to the right of T/2, we discover that they have the same shape as those in part (b) for a_1 but twice as big and negative.

Thus, $b_1 = (-2)\,a_1 = (-2)\,\frac{6}{\pi}\,V = -\frac{12}{\pi}\,V.$

If we calculate a_1, we can use either an integral from T/2 to 3T/4 and double it or we can use an integral from T/2 to T. The latter approach will be used here owing to simpler values substituted into the cos() we get when we integrate $\sin(\omega_0 t)$.

$$b_1 = \frac{2}{T}\int_{T/2}^{T} 6V \sin(\omega_0 t)\,dt$$

or

$$b_1 = \frac{2}{T}(6V)\left[-\frac{\cos(\omega_o t)}{\omega_o}\right]\Bigg|_{T/2}^{T}$$

or

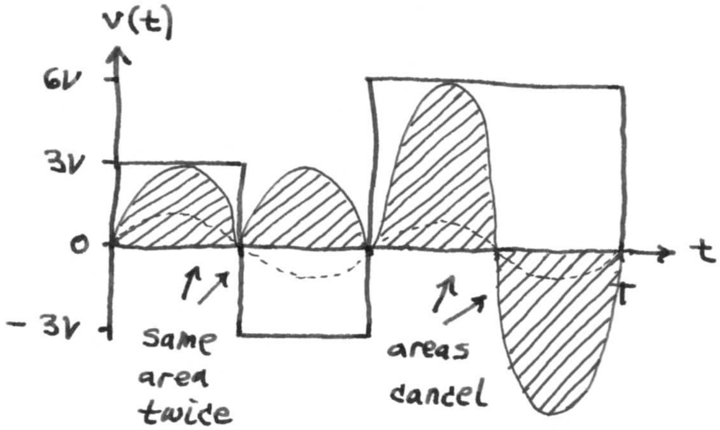
$$b_1 = \frac{12V}{T}\left[\frac{-\cos\left(\frac{2\pi}{T}T\right) - -\cos\left(\frac{2\pi}{T}\frac{T}{2}\right)}{\frac{2\pi}{T}}\right]$$

or

$$b_1 = \frac{6}{\pi}V\ (-1 + -1) = -\frac{12}{\pi}V$$

d) $$b_2 = \frac{2}{T}\int_0^T v(t)\sin(2\omega_o t)\,dt$$

Picture:



We use double the area between 0 and $\frac{T}{4}$.

$$b_2 = \frac{2}{T}(2)\int_0^{T/4} 3V\sin(2\omega_o t)\,dt$$

or

$$b_2 = \frac{4(3V)}{T}\left[-\frac{\cos(2\omega_o t)}{2\omega_o}\right]\Bigg|_0^{T/4}$$

or

$$b_2 = \frac{12V}{T}\ \frac{-\overset{-1}{\cancel{\cos\left(2\left(\frac{2\pi}{T}\right)\frac{T}{4}\right)}} - -\overset{1}{\cancel{\cos\left(2\left(\frac{2\pi}{T}\right)0\right)}}}{2\left(\frac{2\pi}{T}\right)}$$

or

$$b_2 = \frac{3}{\pi}V(2) = \frac{6}{\pi}V$$