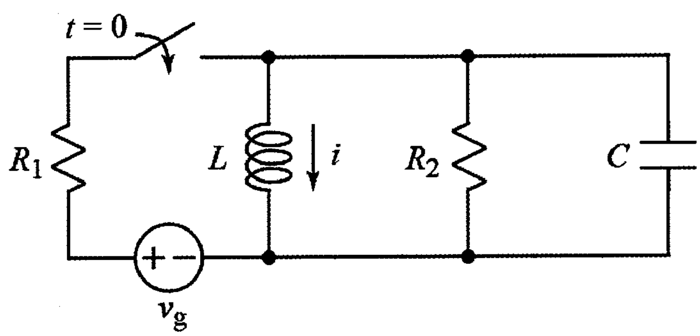
Ex:



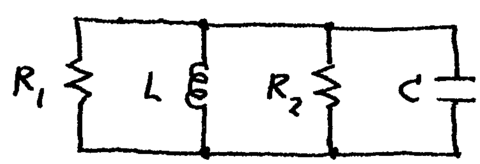
After being open for a long time, the switch closes at $t = 0$.

$v_g = 15$ V $\quad R_1 = 5$ kΩ $\quad R_2 = 15$ kΩ $\quad L = 3.6$ mH $\quad C = 100$ pF

a) State whether $i(t)$ is under-damped, over-damped, or critically-damped.

b) Write a numerical time-domain expression for $i(t)$, $t > 0$, the current through L. This expression must not contain any complex numbers.

sol'n: a) We compute the characteristic roots using the circuit for $t > 0$. We may also may turn off pwr sources to see whether we have a series or parallel RLC circuit:



We may combine $R_1 // R_2 = R$, and we have a parallel RLC.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0^2 = \frac{1}{LC}$$

$$\alpha = \frac{1}{2(5k // 15k)\, 100p} \text{ r/s} = \frac{1}{(2) 5k (1 \| 3)\, 100p}$$

$$\alpha = \frac{1}{(2)\,5k\left(\frac{3}{4}\right)100p}\ r/s = \frac{4}{(3)1000kp}\ r/s = \frac{4}{3}\ M\ r/s$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(3.6m)\,100p} = \frac{1}{0.36p}$$

$$\omega_o^2 = \frac{1}{0.6^2\,\mu^2} = \left(\frac{5}{3}\ M\ r/s\right)^2$$

$$\sqrt{\alpha^2-\omega_o^2} = \sqrt{\left(\frac{4}{3}M\right)^2-\left(\frac{5}{3}M\right)^2} = \sqrt{-\left(\frac{3}{3}M\right)^2}$$

$$\sqrt{\alpha^2-\omega_o^2} = j\ \underbrace{1M\ r/s}_{\omega_d}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2-\omega_o^2} = -\frac{4}{3}M \pm j\,1M\ r/s$$

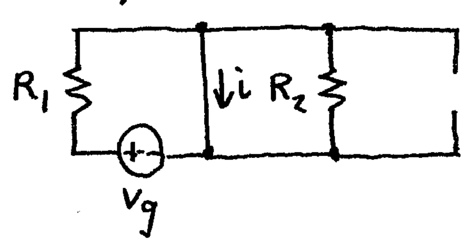
Complex roots ⇒ underdamped

b) For our underdamped roots, our form of sol'n is

$$i(t>0) = A_1 e^{-\alpha t}\cos(\omega_d t) + A_2 e^{-\alpha t}\sin(\omega_d t) + A_3$$

where $\alpha = -\frac{4}{3}M\ r/s$, $\omega_d = 1M\ r/s$

We first find $A_3 = i(t\to\infty)$. As $t\to\infty$, we treat L as wire and C as open:

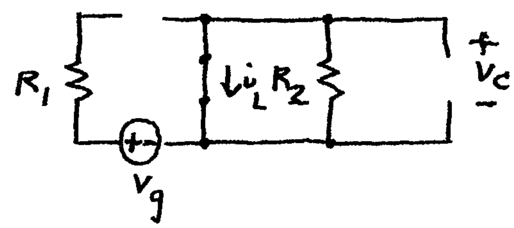


$$i = \frac{v_g}{R_1} = \frac{15V}{5k\Omega}$$

$$i = 3\ mA = A_3$$

Now we match $i(t>0)$ to initial conditions. We start at $t=0^-$ to determine what L and C have as initial conditions.
L = wire, C = open

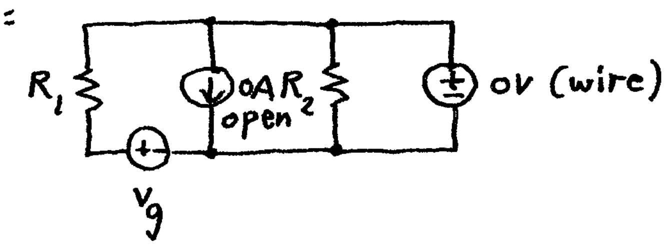
$t=0^-$:

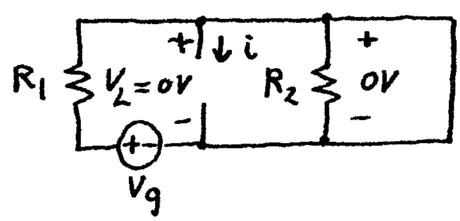


With no power source connected, we have $i_L(0^-) = 0\,A$ and $v_C(0^-) = 0\,V$.

We model the L and C as sources at $t=0^+$.

$t=0^+$:





The wire (the C with 0V) bypasses R_2.

So we have $v_L = 0V$. Since the L is an open, we also have $i(0^+) = 0\,A$.

For our symbolic soln, we have

$$i(0^+) = A_1 + A_3$$

$$i(0^+) = A_1 + A_3 = A_1 + 3\ mA = 0\,A$$

So $A_1 = -3\ mA$

Now we find $\left.\frac{di}{dt}\right|_{t=0^+}$ by writing

$i(t)$ in terms of i_L and/or v_C.

In this case $i(t)$ is i_L.

$$i = i_L$$

We differentiate the eq'n in order to get $\frac{di_L}{dt} = \frac{v_L}{L}$, which is not a derivative.

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \left.\frac{v_L}{L}\right|_{t=0^+} = \frac{0V}{L} = 0\,A/s$$

Our symbolic derivative is

$$\frac{di(0^+)}{dt} = -\alpha A_1 + \omega_d A_2$$

so

$$\left(-\frac{4}{3}M\ r/s\right)\left(-3\,mA\right) + \left(1M\ r/s\right)A_2 = 0\,A/s$$

or

$$A_2 = \frac{-4\,M\,mA}{1M} = -4mA$$

so

$$i(t>0) = -3e^{-\frac{4}{3}Mt}\cos(1Mt) - 4e^{-\frac{4}{3}Mt}\sin(1Mt) + 3 \quad mA$$