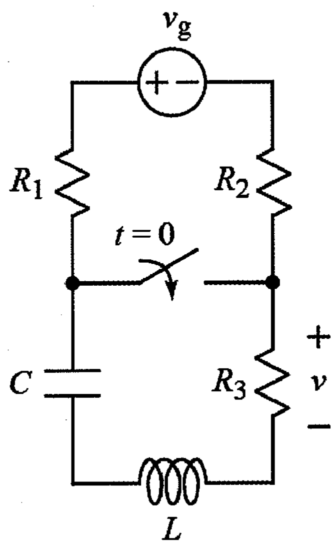
Ex:



After being open for a long time, the switch closes at $t = 0$.

a) Give expressions for the following in terms of no more than v_g, R_1, R_2, R_3, L, and C:

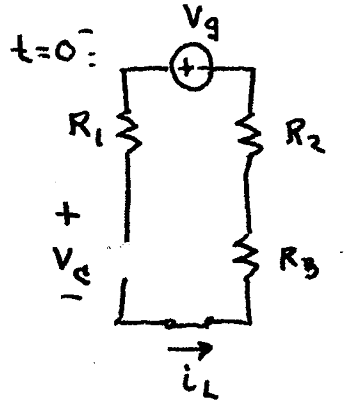
$v(t = 0^+)$ and $\left.\frac{dv(t)}{dt}\right|_{t=0^+}$

b) Find the numerical value of C given the following information:

$s_1 = s_2$ $R_1 = 2.7\ \Omega$ $R_2 = 300\ \Omega$ $R_3 = 2\ \Omega$ $L = 3.3\ \mu H$

sol'n a) To find initial conditions, we first find the value of energy variables i_L and v_C at $t=0^-$. These values will not change as the switch moves. Thus, $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$, and we can model the L and C as an i-src and v-src, respectively.

At $t=0^-$, the circuit has stabilized and L = wire and C = open.

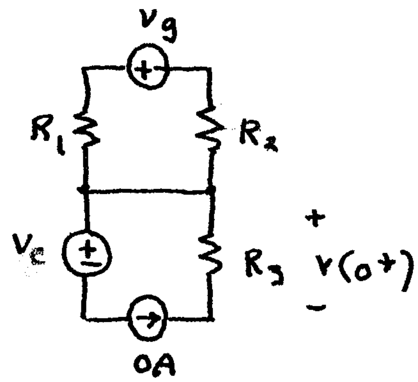


Since C = open, $i_L = 0A$.

No current flows around the loop, so there is no v-drop across the resistors.

Thus, v_g must appear across C. So $v_c = v_g$.

At $t=0^+$, we model the L and C as sources.



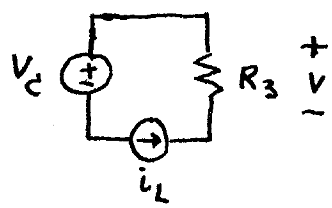
Because of the wire thru the middle, we may ignore the top half of the circuit.

If we consider the bottom half of the circuit, the L looks like an open. Thus, no current flows thru R_3, and there is no v-drop across R_3.

Thus, $v(0^+) = 0A$.

To find $\left.\frac{dv}{dt}\right|_{t=0^+}$, a standard approach is to first write a circuit eq'n relating v to energy vars i_L and v_c. Then we will be able to differentiate the eq'n and convert $\frac{di_L}{dt}$ to $\frac{v_L}{L}$ and $\frac{dv_c}{dt}$ to $\frac{i_c}{C}$.

It is easier to write an eq'n for v in terms of i_L and v_C if we replace the L and C with sources called i_L and v_C. Again, we may use only the bottom half of the circuit.



We see that i_L flows thru R_3 (but in the opposite direction from the passive sign convention.

$$v = -i_L R$$

Differentiate the eq'n:

$$\frac{dv}{dt} = -\frac{di_L}{dt} \cdot R = -\frac{v_L}{L} \cdot R$$

Now, (after differentiating), evaluate at $t = 0^+$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = -\frac{v_L(0^+)R}{L}$$

Earlier, in our $t=0^+$ circuit, we found $v(0^+) = 0$. So for a v-loop around the bottom of the circuit will give the result that $v_L = -v_g$.

$$\left.\frac{dv}{dt}\right|_{t=0^+} = --\frac{v_g R}{L} = v_g \frac{R}{L}$$

b) If $s_1 = s_2$, then we have critical damping, so $s_1 = s_2 = -\alpha$, $\sqrt{\alpha^2 - \omega_0^2} = 0$

After $t=0$, the circuit (lower half) is a series RLC.

$$\alpha = \frac{R}{2L} \qquad \omega_0^2 = \frac{1}{LC}$$

So we have $\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = 0.$

or $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

or $\frac{R^2}{4L^2} = \frac{1}{LC}$

We can turn the two sides upside down.

$$\frac{4L^2}{R^2} = LC$$

or

$$C = \frac{4L^2}{R^2 L} = \frac{4L}{R^2} = \frac{4 \cdot 3.3\mu F}{2^2}$$

(we use $R = R_3 = 2\Omega$.)

$$C = 3.3\,\mu F$$