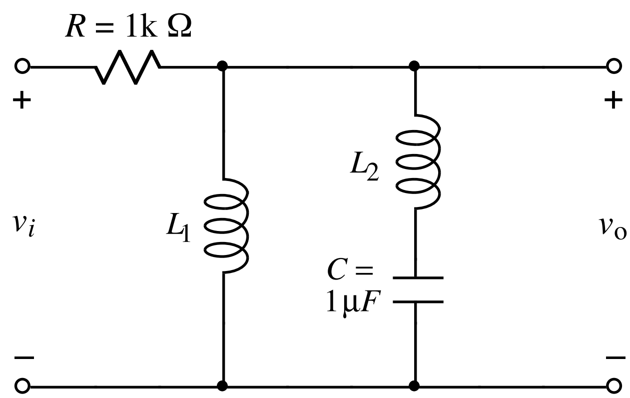
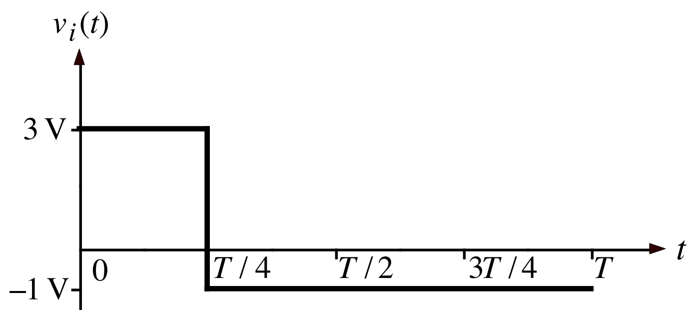
Ex:





T = one period of $v_i(t) = 2\pi$ ms

$$v_i(t) = \begin{cases} 3\text{ V} & 0 < t \le T/4 \\ -1\text{ V} & T/4 < t \le T \end{cases}$$

a) Find values of $L_1 \neq 0$ and $L_2 \neq 0$ for the above filter circuit such that the magnitude of the transfer function equals one for the fundamental and zero for the second harmonic of $v_i(t)$, also shown above.

b) Find numerical values of coefficients a_v, a_1, and b_1 for the Fourier series for $v_i(t)$:

$$v_i(t) = a_v + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

soln: a) For the circuit configuration given in this problem, we have $|H|=0$ when L_2 and C sum to $0\,\Omega$, and we have $|H|=1$ when L_1 in parallel with L_2, C forms an open circuit.

In other words, $j\omega L_2 + \frac{1}{j\omega C} = 0$ @ $2\omega_0$.

Since $T = 2\pi$ ms, $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi \text{ ms}} = 1k$ r/s.

So we want $j\omega L_2 + \frac{1}{j\omega C} = 0$ @ 2k r/s

or $(2\omega_0)^2 = \frac{1}{L_2 C} = (2k)^2$

or $L_2 = \frac{1}{(2k)^2 \cdot C} = \frac{1}{(2k)^2\, 1\mu} H = \frac{1}{4} H$

$$\boxed{L_2 = 250 \text{ mH}}$$

At ω_0 we have $j\omega_0 L_2 + \frac{1}{j\omega_0 C} = j1k \cdot \frac{1}{4} + \frac{1}{j1k1\mu}$

$= j250 - j1k$

$= -j750\,\Omega$

We want $j\omega L_1 = j750\,\Omega$

or $j1k\, L_1 = j750\,\Omega$

or $L_1 = \frac{3}{4} H = 750$ mH

$$\boxed{L_1 = 750 \text{ mH}}$$

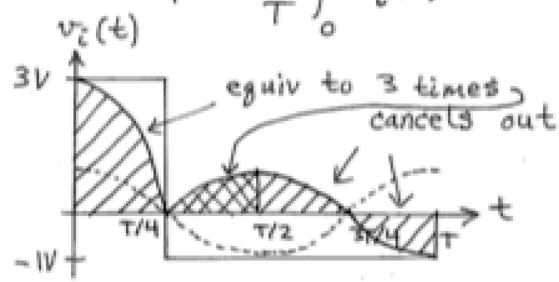
b) To find a_v, we note that the area on the left side is $3V \cdot \frac{T}{4}$ and the negative area on the right side is $-1V \cdot \frac{3T}{4} = -3V \cdot \frac{T}{4}$. So the areas are equal but opposite and cancel out to give.

$$a_v = 0\ V$$

(The formal calculation is $a_v = \frac{1}{T}\int_0^T v_i(t)dt$.)

To find a_1 we graphically analyze the area of the integral for the coefficient:

$$a_1 = \frac{2}{T}\int_0^T v_i(t)\cos(\omega_o t)\,dt$$



The net area under the curve for $v_i(t)\cos(\omega_o t)$ is

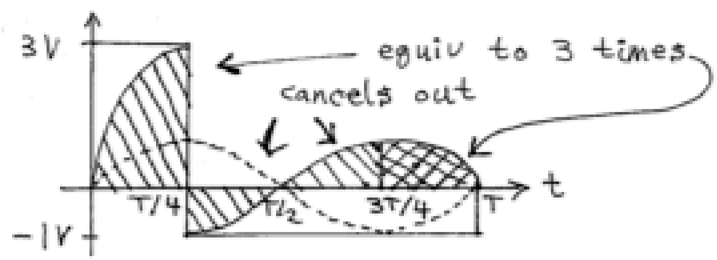
$$4\int_{T/4}^{T/2} -1\cdot\cos(\omega_o t)\,dt \quad \text{(4 copies of region)}$$

$$= -4\left.\frac{\sin(\omega_o t)}{\omega_o}\right|_{T/4}^{T/2}$$

$$= \frac{-4}{\frac{2\pi}{T}}\left(\sin\pi - \sin\pi/2\right) = +\frac{2T}{\pi}$$

Thus $a_1 = \frac{2}{T}\cdot\frac{T}{\pi}\ 2 = \frac{4}{\pi}\ V$

For b_1 we find the area of $v_i(t)\sin(\omega_0 t)$



We can calculate the area as 4 times the ▩ region on the right.
But the area of the ▩ is the same as the ▩ region for a_1, above.

$$\text{So } b_1 = a_1 = \frac{4}{\pi} \text{ V.}$$