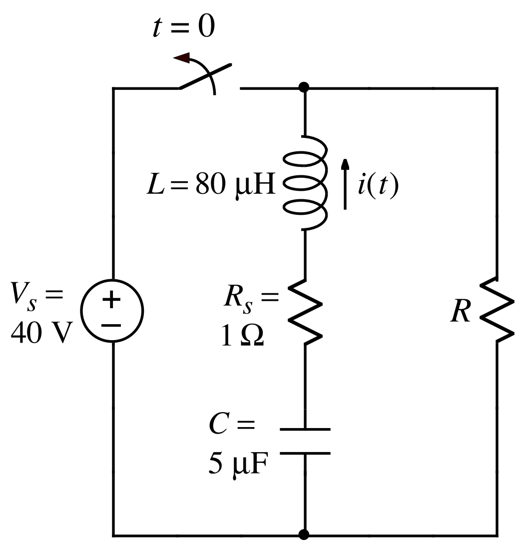
HOMEWORK #9 probs 1 and 2 solution

Ex:



After being closed for a long time, the switch opens at $t = 0$.

The above circuit is an analog "one-shot" circuit that, once charged, produces a short, rounded current-pulse resembling the current that flows in a synapse of a neuron. The circuit is critically damped.

a) Find the value of R that makes the circuit critically-damped.

b) Using the R value from (a), find a numerical expression for the inductor current, $i(t)$, for $t > 0$.

sol'n: a) After $t=0$, the switch is open, and the circuit becomes a series RLC with resistance $R+R_s$.

Characteristic roots are (always)

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $\alpha = \frac{R}{2L}$ for series RLC

$\omega_0^2 = \frac{1}{LC}$ for series or parallel RLC

For critically-damped, we have

$$s_1 = s_2 \Rightarrow \alpha = \omega_0 \quad \text{or} \quad \alpha^2 = \omega_0^2$$

or

$$\left(\frac{R_{eq}}{2L}\right)^2 = \frac{1}{LC} \quad \text{with} \quad R_{eq} = R_s + R$$

or

$$R_{eq}^2 = \frac{2^2 L}{C}$$

or

$$R_{eq} = 2\sqrt{\frac{L}{C}} = 2\cdot\sqrt{\frac{80\mu}{5\mu}}\,\Omega = 8\,\Omega$$

or

$$1\,\Omega + R = 8\,\Omega$$

or

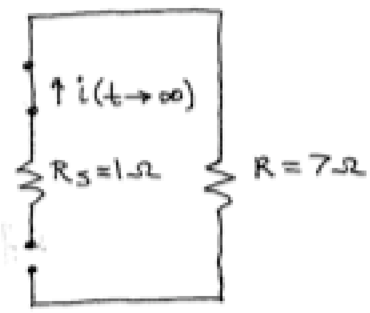
$$R = 7\,\Omega$$

b) We use the general form of sol'n for critically damped:

$$i(t>0) = A_1 e^{st} + A_2 t e^{st} + A_3$$

We find A_3 from $i(t\to\infty)$.

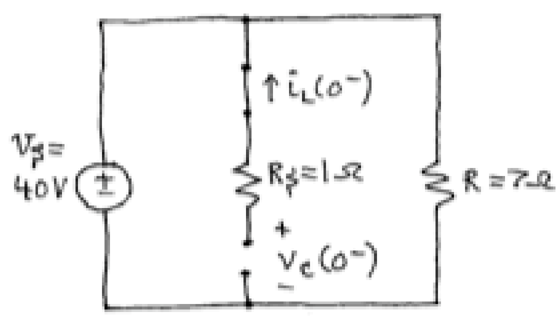
$t \to \infty$: C = open, L = wire, switch open



Since there is no pwr source,
$A_3 = i(t \to \infty) = 0$ A

We find A_1 and A_2 from initial cond's:

$t = 0^-$: C = open, L = wire, switch closed

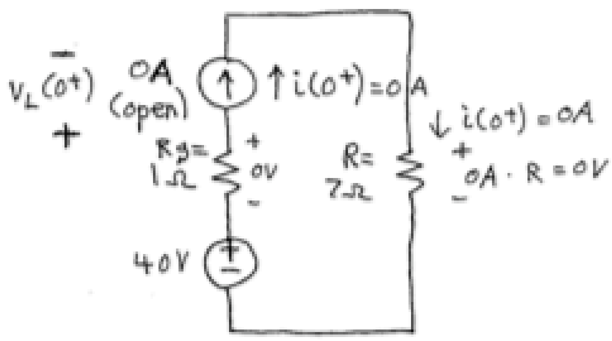


$i_L(0^-) = 0A$ (no path for current)

$v_c(0^-) = 40V$ since no current in R_g means 0V across R_g and v-loop on left must sum to zero volts.

We find $i(0^+)$ and $\left.\dfrac{di(t)}{dt}\right|_{t=0^+}$.

$t=0^+$: $\quad i_L(0^+) = i_L(0^-) = 0A, \; v_C(0^+) = v_C(0^-) = 40V$



$$i(0^+) = i_L(0^+) = 0A$$

For the symbolic sol'n we have

$$i(0^+) = A_1$$

Equating the circuit value and symbolic sol'n form, we have $A_1 = 0$ A.

For the derivative, we have

$$\left.\frac{di(t)}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L}$$

Using a v-loop, we have $v_L(0^+) = 40V$.

For the symbolic sol'n we have

$$\left.\frac{di(t)}{dt}\right|_{t=0^+} = \cancelto{0}{A_1} s + A_2 = A_2 = \frac{40V}{80\mu}$$

Thus, $i(t>0) = 500k\,t\,e^{st}$ A

where $s = \frac{R_{eq}}{2L} = \frac{8}{2(80\mu)} = 50k$ r/s.