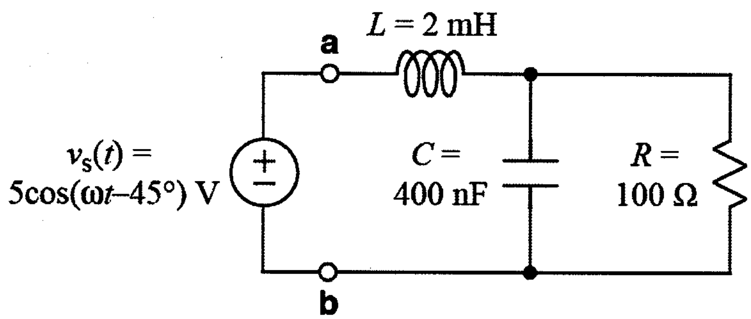
HOMEWORK #7 prob 1 solution

Ex:



Note: $\omega = 25$ k r/s.

Do the following for the impedance to the right of the a, b terminals:

a) Calculate complex power $S = P + jQ$.

b) Calculate average (or DC) power.

c) Calculate maximum instantaneous power.

d) Sketch the power waveform, $p(t)$.

soln: a) We calculate total impedance to right of a, b so we can use the following formula for S:

$$S = \frac{|V|^2}{2Z^*} \quad \text{where } V = V_S$$

Total Z:

$$Z = j\omega L + R \parallel \frac{1}{j\omega C}$$

$$j\omega L = j\,25k \cdot 2m\,\Omega = j\,50\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{25k \cdot 400n}\,\Omega = \frac{-j}{10\,k\,kn} = -j\,100\,\Omega$$

$$Z = j\,50\,\Omega + 100 \parallel -j\,100\,\Omega$$

$$= j\,50\,\Omega + 100\,\Omega \cdot 1 \parallel -j$$

$$= j\,50\,\Omega + 100 \cdot \frac{-j}{1-j}$$

$$z = j50\,\Omega + 100 \cdot \frac{-j}{1-j} \, \frac{1+j}{1+j}\,\Omega$$

$$= j50\,\Omega + 100 \, \frac{1-j}{2}\,\Omega$$

$$= j50\,\Omega + 50\,\Omega - j50\,\Omega$$

$$z = 50\,\Omega$$

Thus, $$S = \frac{|V_S|^2}{2 \cdot Z^*} = \frac{5^2}{(2)50\,\Omega} = \frac{1}{4}\ \text{VA}$$

Note: Because z is real, $z^* = z$ and S is real.

b) $S = P + jQ$ so $P = \text{Re}[S]$, $Q = \text{Im}[S]$.

$P = \frac{1}{4}$ W Note: $Q = 0$ here.

c) The max instantaneous power is the DC power level, P, plus the magnitude of the sinusoidal part of the power waveform, $|S|$.

$$\max_t p(t) = P + |S| = P + P = \frac{1}{2}\,W$$

d) Since $S = P$ is real, the phase shift of the sinusoidal part of $p(t)$ has zero phase shift.