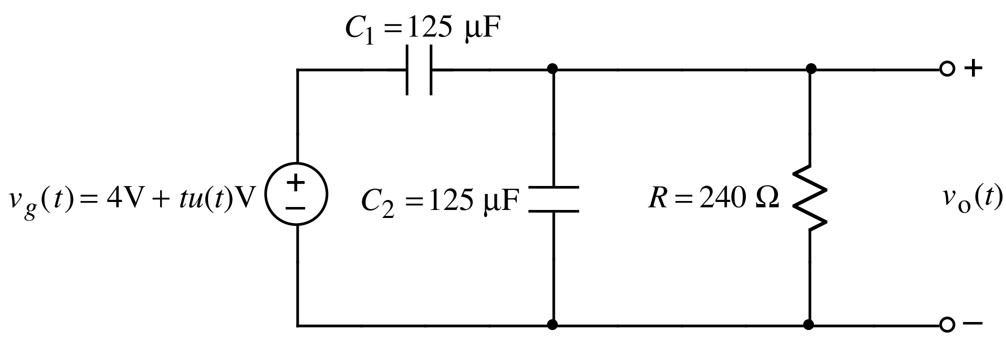
HOMEWORK #6 prob 2, 3 solution

Ex:



Note: The 4 V in the $v_g(t)$ source is always on.

a) Write the Laplace transform, $V_g(s)$, of $v_g(t)$.

b) Draw the s-domain equivalent circuit, including source $V_g(s)$, components, initial conditions for C's, and terminals for $V_o(s)$.

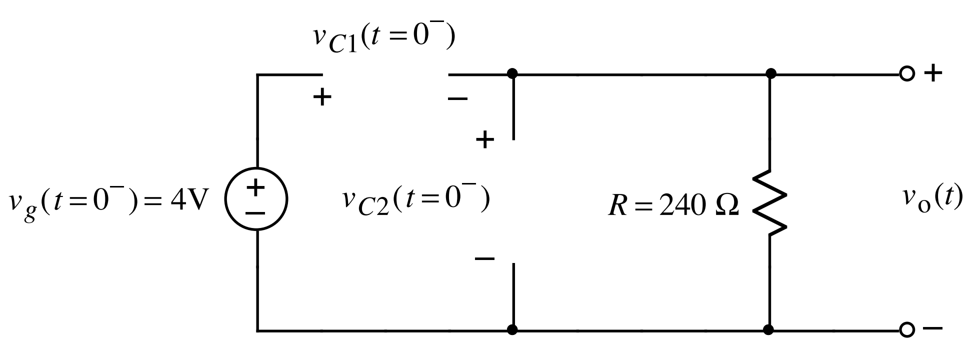
c) Write an expression for $V_o(s)$.

d) Apply the initial value theorem to find $\lim_{t \to 0^+} v_o(t)$.

SOL'N: a) We consider only the value of $v_g(t)$ for $t > 0$ when finding the Laplace transform:

$$\mathcal{L}\{v_g(t)\} = \mathcal{L}\{4 + tu(t)\}\text{V} = \frac{4}{s} + \frac{1}{s^2}\text{V}$$

b) To find initial conditions, we assume that, since the circuit input is a constant 4 V, the circuit has reached a constant state where derivatives of voltages and currents are zero. Thus, we treat the capacitors as open circuits. We then find the energy variables, $v_{C1}(t = 0^+)$ and $v_{C2}(t = 0^+)$:

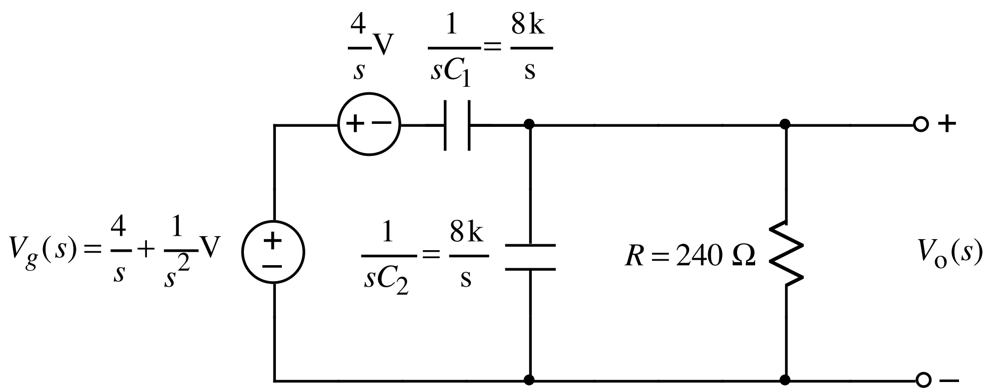


The R discharges C_2 so all of the source voltage appears across C_1:

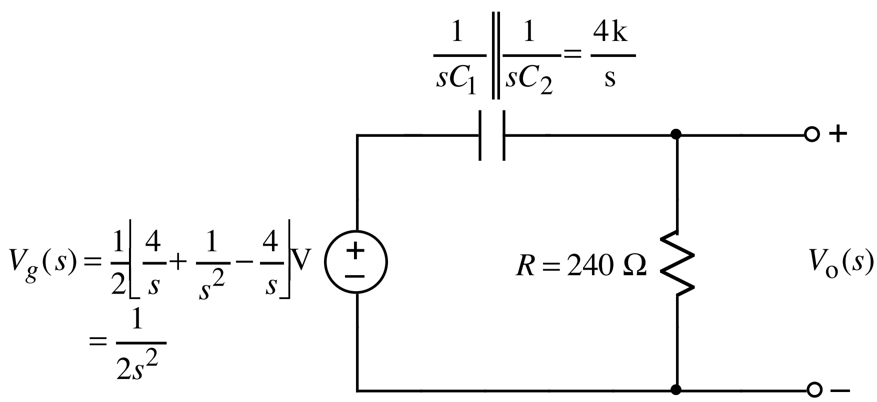
$$v_{C1}(0^-) = 4\,\mathrm{V}$$

$$v_{C2}(0^-) = 0\,\mathrm{V}$$

We have a choice of whether to use a current source or a voltage source for the initial conditions on C_1. (We may omit the initial condition source for C_2, since the initial value is zero.) The choice made here is to use a series voltage sourse. Note that the voltage source corresponds to a step function in the time domain that produces voltage $v_{C1}(0^-)$ in the desired direction.



c) We sum the voltage sources and then convert the voltage sources, C_1, and C_2 to a Thevenin equivalent form.



$$V_o(s) = \frac{1}{2s^2}\frac{240}{240 + \frac{4\text{k}}{\text{s}}} = \frac{240}{s(480s + 8\text{k})}$$

d) The initial value theorem statement is as follows:

$$\lim_{t\to 0^+} v_o(t) = \lim_{s\to\infty} sV_o(s)$$

or, in this case,

$$\lim_{t\to 0^+} v_o(t) = \lim_{s\to\infty} s\left(\frac{240}{s(480s + 8\text{k})}\right) = \lim_{s\to\infty} \frac{240s}{480s^2} = 0.$$

The highest power of s in the denominator is larger than the highest power of s in the numerator. Thus, the value is zero.

This result makes sense, since C_2 has zero volts across it at time $t = 0^+$, and the change in the input signal is zero at time $t = 0^+$.