EX: Find the inverse Laplace transform of $\dfrac{e^{-2s}}{(s+4)^n}$.

SOL'N: The e^{-2s} indicates that we have a delayed signal, for which we use the delayed signal (or shifted signal) identity:

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s), \qquad a > 0$$

We may apply this identity at the end of our calculations. Thus, we wish to find the inverse Laplace transform of

$$F(s) = \frac{1}{(s+4)^n}$$

We may employ the following transform pair:

$$\mathcal{L}\{t^n e^{-at}\} = \frac{n!}{(s+a)^{n+1}}$$

There is a minor issue with the exponent being $n + 1$ in the denominator, which we may resolve by using $n - 1$ on the left side instead of n:

$$\mathcal{L}\{t^{n-1} e^{-at}\} = \frac{(n-1)!}{(s+a)^n}$$

We may also divide by $(n - 1)!$ to obtain the precise form we need for the present situation:

$$\mathcal{L}\left\{\frac{t^{n-1}e^{-at}}{(n-1)!}\right\} = \frac{1}{(s+a)^n}$$

For $a = 4$, we obtain the following result:

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+4)^n}\right\} = \frac{t^{n-1}e^{-4t}}{(n-1)!}$$

Now we apply the delay identity with $a = 2$, which means we replace t with $t - 2$ wherever t appears:

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{(s+4)^n}\right\} = \frac{(t-2)^{n-1}e^{-a(t-2)}}{(n-1)!}u(t-2)$$