Ex: Find the inverse Laplace transform for the following expression:

$$F(s) = -\frac{-s^3 - 36s^2 - 863s - 6900}{(s^2 + 24s + 400)(s^2 + 24s + 225)}$$

SOL'N: Factor the denominator to find the "partial fraction" terms needed. (We actually write terms that represent decaying cosines and sines.)

$$F(s) = \frac{s^3 + 36s^2 + 863s + 6900}{[(s+12)^2 + 16^2][(s+12)^2 + 9^2]}$$

$$= \frac{A(s+12)}{(s+12)^2 + 16^2} + \frac{B(16)}{(s+12)^2 + 16^2} + \frac{C(s+12)}{(s+12)^2 + 9^2} + \frac{D(9)}{(s+12)^2 + 9^2}$$

or

$$F(s) = \frac{A(s+12) + B(16)}{(s+12)^2 + 16^2} + \frac{C(s+12) + D(9)}{(s+12)^2 + 9^2}$$

We can use a common denominator and then solve for the coefficients.

$$F(s) = \frac{[A(s+12) + B(16)][(s+12)^2 + 9^2]}{[(s+12)^2 + 16^2][(s+12)^2 + 9^2]}$$

$$+ \frac{[C(s+12) + D(9)][(s+12)^2 + 16^2]}{[(s+12)^2 + 16^2][(s+12)^2 + 9^2]}$$

The multipliers for each power of s are as follows:

$$s^3 : A + C$$

$$s^2 : 12A + 24A + 16B + 12C + 24C + 9D$$

$$s : 225A + 24(16)B + 24(12)A + 400C + 24(9)D + 24(12)C$$

$$1 : (12A + 16B)225 + (12C + 9D)400$$

Equating the above values with the numerator coefficients in the original $F(s)$, we work through the four equations.

$$A + C = 1$$

$$36(A + C) + 16B + 9D = 36$$

$$225A + 24(16)B + 24(12)(A + C) + 400C + (24)9D = 863$$

$$12(225)A + 225(16)B + 12(400)C + 400(9)D = 6900$$

or

$$A + C = 1$$
$$36 + 16B + 9D = 36$$
$$225A + 24(16)B + 24(12) + 400C + (24)9D = 863$$
$$12(225)A + 225(16)B + 12(400)C + 400(9)D = 6900$$

or

$$A + C = 1$$
$$16B + 9D = 0$$
$$225A + 24(16B + 9D) + 400C = 863 - 24(12)$$
$$12(225)A + 225(16)B + 12(400)C + 400(9)D = 6900$$

or

$$A + C = 1$$
$$16B + 9D = 0$$
$$225A + 400C = 575$$
$$12(225)A + 225(16)B + 12(400)C + 400(9)D = 6900$$

or

$$A + C = 1$$
$$16B + 9D = 0$$
$$9A + 16C = 23$$
$$12(225)A + 225(16)B + 12(400)C + 400(9)D = 6900$$

From the first and third equations, we find the following:

$$A = -1$$
$$C = 2$$

We are left with the following equations:

$$16B + 9D = 0$$
$$\frac{1}{25}[12(225)(-1) + 225(16)B + 12(400)2 + 400(9)D = 6900]$$

or

$$16B + 9D = 0$$
$$12(9)(-1) + 9(16)B + 12(16)2 + 16(9)D = 276$$

or

$$16B + 9D = 0$$
$$9(16)B + 16(9)D = 0$$

or

$$16B + 9D = 0$$
$$B + D = 0$$

or

$$B = 0$$
$$D = 0$$

Summary:

$$F(s) = \frac{(-1)(s+12)}{(s+12)^2 + 16^2} + \frac{2(s+12)}{(s+12)^2 + 9^2}$$

Now we take the inverse transform of the above to get our final result.

$$f(t) = [-e^{-12t}\cos(16t) + 2e^{-12t}\cos(9t)]u(t)$$

NOTE: We multiply by $u(t)$ as a reminder that we are uncertain of the value of $f(t)$ for $t < 0$.