

Ex: Find the Laplace transforms of the following waveforms:

$$t \int_0^t e^{-at} dt$$

SOL'N: Working from the inside out, we first apply the identity for integration:

$$\mathcal{L} \left\{ \int_0^t f(t) dt \right\} = \frac{F(s)}{s}$$

where

$$F(s) = \mathcal{L} \{ e^{-at} \} = \frac{1}{s+a}$$

$$\text{Thus, } \mathcal{L} \left\{ \int_0^t e^{-at} dt \right\} = \frac{1}{s(s+a)}$$

Now we apply the identity for multiplication by t :

$$\mathcal{L} \{ tf(t) \} = -\frac{d}{ds} F(s)$$

$$\text{Thus, } \mathcal{L} \left\{ t \int_0^t e^{-at} dt \right\} = -\frac{d}{ds} \frac{1}{s(s+a)}$$

$$= \frac{1}{s^2(s+a)} + \frac{1}{s(s+a)^2}$$