

EX: Find the Laplace transforms of the following waveform:

$$t \frac{d}{dt} [te^{-at}]$$

SOL'N: We work from the inside out.

From a table of transform pairs,

$$\mathcal{L} \{ te^{-at} \} = \frac{1}{(s+a)^2}$$

Now we apply the identity for derivatives:

$$\mathcal{L} \left\{ \frac{d}{dt} f(t) \right\} = sF(s) - f(0^-)$$

Thus,

$$\mathcal{L} \left\{ \frac{d}{dt} [te^{-at}] \right\} = s \cdot \frac{1}{(s+a)^2} - te^{-at} \Big|_{t=0^-}$$

$$= \frac{s}{(s+a)^2}$$

Finally, we apply the identity for multiplication by t :

$$\mathcal{L} \{ t f(t) \} = -\frac{d}{ds} F(s)$$

In the present case, we have

$$\mathcal{L} \left\{ t \frac{d}{dt} [te^{-at}] \right\} = -\frac{d}{ds} \frac{s}{(s+a)^2}$$

$$= \frac{-1}{(s+a)^2} + \frac{2s}{(s+a)^3}$$