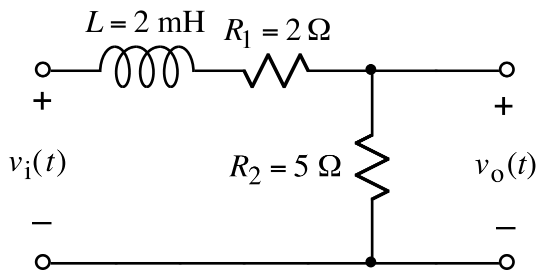
HOMEWORK #4 probs 4 and 5 solution

Ex:



a) For the above circuit, determine the transfer function $H(j\omega) = V_o/V_i$.

b) Assume the circuit in problem 4, has the following input signal:

$$v_i(t) = 8 + \sum_{k=1}^{\infty} \frac{36}{k^2}\left[(2k-1)\cos(k\omega_0 t) - 2\sin(k\omega_0 t)\right] \text{ V}$$

Note: $\omega_o = 2$ k rad/s for the Fourier series.
Write the time-domain expression of the sixth harmonic (i.e., $k = 6$) of $v_o(t)$.

sol'n: The sixth harmonic of $v_o(t)$ arises solely from the sixth harmonic of $v_i(t)$ owing to the property of sinusoidal inputs producing only sinusoidal signals of the same frequency everywhere in the circuit. Thus, we focus on $v_{i6}(t)$:

$$v_{i6}(t) = \frac{36}{6^2}\left[(2(6)-1)\cos(6\omega_o t) - 2\sin(6\omega_o t)\right] \text{V}$$

or

$$v_{i6}(t) = 11\cos(12\text{k r/s} \cdot t) - 2\sin(12\text{k r/s}\, t) \text{V}$$

We find the output $v_{o6}(t)$ for the circuit when $v_{i6}(t)$ is the input. The phasor of the input signal is

$$V_{i6} = 11 + j2.$$

We have a V-divider:

$$V_{o6} = V_{i6} \cdot \frac{R_2}{R_1 + R_2 + j\omega L} \quad \text{where } \omega = 6\omega_o$$

or

$$V_{o6} = H(j6\omega_o)V_{i6} \quad \text{where} \quad H(j6\omega_o) = \frac{R_2}{R_1+R_2+j6\omega_o L}$$

The impedance of L is

$$j\omega L = j6\omega_o L = j6 \cdot 2\text{k r/s} \cdot 2\text{mH} = j24\ \Omega.$$

Our transfer function is

$$H(j6\omega_o) = \frac{5\Omega}{2\Omega + 5\Omega + j24\Omega} = \frac{5}{7+j24}$$

Combining results yields the value of V_{o6}:

$$V_{o6} = H(j6\omega_o)V_{i6} = \frac{5}{7+j24}(11+j2)\ \text{V}$$

To simplify this complex value, we rationalize:

$$V_{o6} = \frac{5}{7+j24}(11+j2) \cdot \frac{7-j24}{7-j24} = \frac{5[(77+48)-j(264-14)]}{7^2+24^2}\ \text{V}$$

or

$$V_{o6} = \frac{5(125-j250)}{25^2}\ \text{V}$$

or

$$V_{o6} = 1-j2\ \text{V}$$

Converting back to the time domain (and remembering that the inverse phasor of $-j$ is $\sin(\omega t)$, we have the time domain expression for the sixth harmonic of $v_o(t)$:

$$v_{o6}(t) = \cos(12\text{k r/s} \cdot t) + 2\sin(12\text{k r/s} \cdot t)\ \text{V}$$