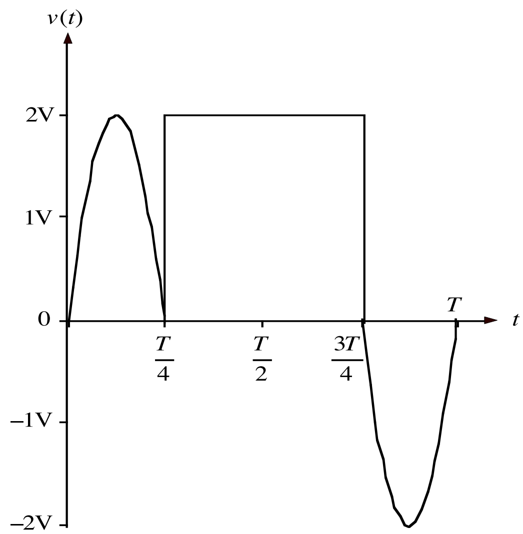
# HOMEWORK #4 probs 2 and 3 solution

Ex:




One period, $T$, of a function $v(t)$ is shown above. The formula for $v(t)$ is

$$v(t) = \begin{cases} 2\sin\left(\frac{4\pi}{T}t\right)\text{ V} & 0 < t < T/4 \\ 2\text{V} & T/4 < t < 3T/4 \\ 2\sin\left(\frac{4\pi}{T}t\right)\text{ V} & 3T/4 < t < T \end{cases}$$

Find the numerical value of the following coefficients of the Fourier series for $v(t)$:
(Hint: separate the function into even and odd parts.)

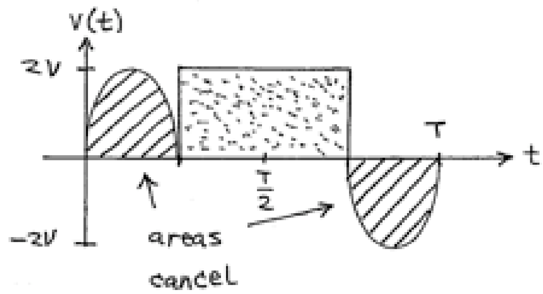
a) $a_v$ b) $a_1$

c) $b_2$ d) $a_4$

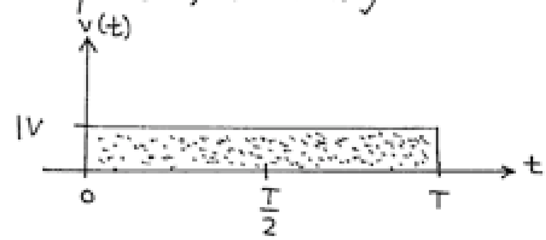
sol'n: a) When calculating $a_\nu$ as the area of one period of $v(t)$ divided by the period, i.e.,

$$a_\nu = \frac{1}{T}\int_0^T v(t)\,dt,$$

we observe that the areas of the sinusoidal portions cancel out.




We spread the remaining rectangle across one period, resulting in a height of 1V.



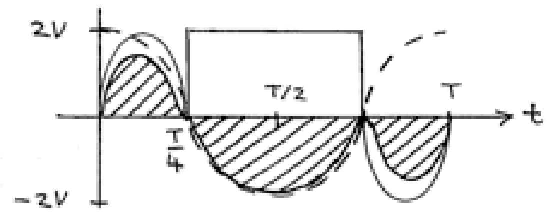

$a_\nu = 1V$ (average height of $v(t)$)

b) The $a_1$ coefficient formula is

$$a_1 = \frac{2}{T}\int_0^T v(t)\cos(\omega_0 t)\,dt$$

where $\omega_0 = \frac{2\pi}{T}$

Graphically, the product in the integral gives the following areas (shown by hatched lines) when the integral is computed:




The first and last areas cancel, and we may double the first half of the center area to compute $a_1$:

$$a_1 = \frac{2}{T}\cdot 2\int_{T/4}^{T/2} 2V\cdot\cos\left(\frac{2\pi}{T}t\right)dt$$

or

$$a_1 = \frac{8V}{\cancel{T}}\left.\frac{\sin\left(\frac{2\pi}{T}t\right)}{\frac{2\pi}{\cancel{T}}}\right|_{T/4}^{T/2}$$

or

$$a_1 = \frac{4}{\pi}V\cdot\left(\underbrace{\sin\pi}_{=0} - \underbrace{\sin\frac{\pi}{2}}_{=1}\right)$$

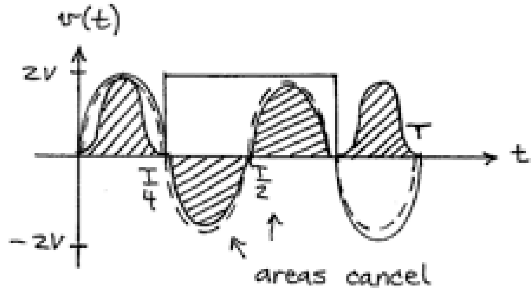
or

$$a_1 = -\frac{4}{\pi}V$$

c) The $b_2$ coefficient formula is

$$b_2 = \frac{2}{T}\int_0^T v(t)\sin(2\omega_0 t)\,dt, \qquad \omega_0 = \frac{2\pi}{T}$$

We may determine the value of the integral graphically by finding the area under $v(t)\cdot\sin(2\omega_0 t)$.

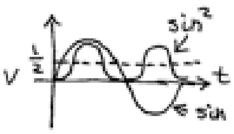



The first and last areas are the same, and the middle two areas cancel.

$$b_2 = \frac{2}{T}\cdot 2\int_0^{T/4} 2\sin\left(\frac{4\pi}{T}t\right)\sin\left(\frac{4\pi}{T}t\right)dt \quad V$$

$$= \frac{8}{T}\int_0^{T/4}\sin^2\left(\frac{4\pi}{T}t\right)dt \ V$$

$$= \frac{8}{T}\int_0^{T/4}\frac{1}{2} - \frac{1}{2}\cos\left(\frac{8\pi}{T}t\right)dt \ V$$




$$= \frac{8}{T}\left(\frac{1}{2}t - \frac{\frac{1}{2}\sin\frac{8\pi}{T}t}{\frac{8\pi}{T}}\right)\Bigg|_0^{T/4} V$$

$\sin^2 = \frac{1}{2} - \frac{1}{2}\cos(2\text{-freq})$

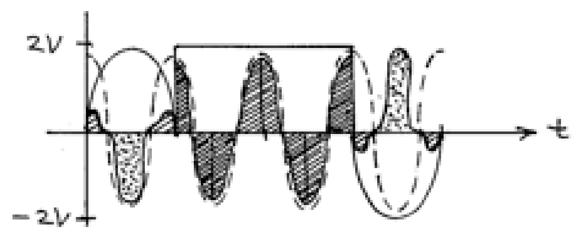
$$= \frac{8}{T}\cdot\frac{T}{8} - \frac{1}{2\pi}(\sin 2\pi - 0)\ V$$

$$b_2 = 1\ V$$

d) The $a_4$ coefficient formula is

$$a_4 = \frac{2}{T}\int_0^T v(t)\cos(4\omega_0 t)\,dt$$

The integral is the area under the product of $v(t)$ and $\cos(4\omega_0 t)$:




The areas all cancel out.

$$a_4 = 0\text{ V}$$

Note: The preceding results may also be obtained by breaking $v(t)$ into even and odd parts. The even part gives $a_v$ and $a_1$ and $a_4$. The odd part gives $b_2$. $v(t) = v_{even}(t) + v_{odd}(t)$