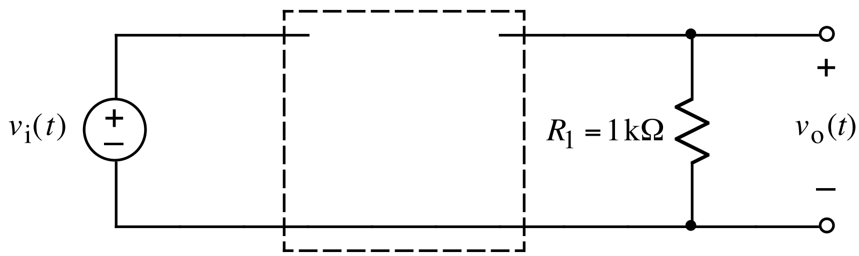
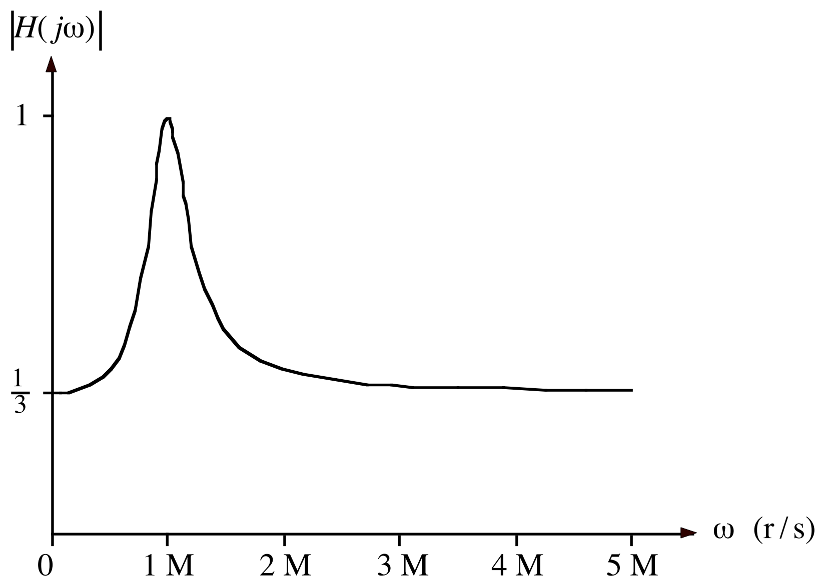
ECE 2260
F 10

HOMEWORK #4 prob 1 solution

Ex:





Given the resistor connected as shown and using not more than one each R, L, and C in the dashed-line box, design a circuit to go in the dashed-line box that will produce the **band-pass** |H($j\omega$)| vs. ω shown above. That is:

$$\max_{\omega}|H(j\omega)| = 1 \text{ and occurs at } \omega_0 = 1 \text{ M r/s}$$

$$|H(j\omega)| = \frac{1}{3} \text{ at } \omega = 0 \qquad \text{and} \qquad \lim_{\omega\to\infty}|H(j\omega)| = \frac{1}{3}$$

If you use a C in your solution, use C = 0.5 nF.

sol'n: To achieve a gain of one, i.e., $|H| = 1$, at $\omega_o = 1$ Mr/s we need to satisfy one of two following conditions:

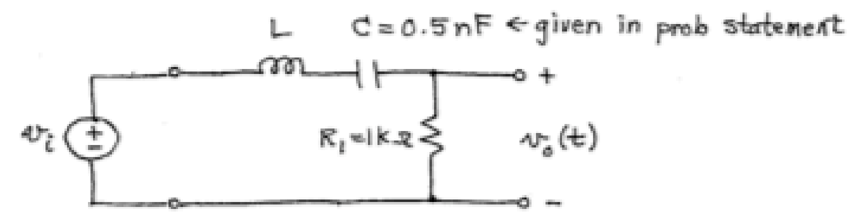
1) There is an open circuit from the top rail to the bottom rail when $\omega = \omega_o$,

or 2) There is a short circuit in the top rail connecting the input directly to the output when $\omega = \omega_o$.

Given the $1k\Omega$ resistor on the right side, only condition (2) is possible.

To achieve the short circuit at $\omega_o = 1$ Mr/s, we use a series L and C.

The circuit thus far:



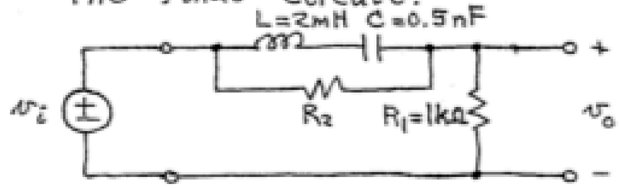
To find L, we have $\omega_o^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega_o^2 C}$

$$L = \frac{1}{(1M)^2 \frac{1}{2}n} \text{ H} = 2 \text{ mH}$$

The above circuit is incomplete, as $H(j\omega)$ will be zero at $\omega = 0$, (when the C becomes an open circuit), and $H(j\omega)$ will be zero as $\omega \to \infty$, (when the L becomes an open circuit).

To achieve a gain of 1/3, i.e. $|H| = \frac{1}{3}$, at $\omega = 0$ and $\omega \to \infty$ the L and C must be bypassed by a resistor.

The final circuit:



At $\omega = 0$ and $\omega \to \infty$, the circuit effectively consists of the v_i source and the two R's. Using the voltage-divider formula, we have (for $\omega = 0$ or $\omega \to \infty$)

$$|H(j\omega)| = \frac{1}{3} = \frac{R_1}{R_1 + R_2}.$$

This means R_1 is $\frac{1}{3}$ of the total of $R_1 + R_2$.

In other words, the ratio of R_1 to the total of $R_1 + R_2$ is 1 to 3. It follows that $R_1 + R_2 = 3k\Omega$, since $R_1 = 1k\Omega$.

Thus, $R_2 = 2\,k\Omega$.

Note: Does R_2 alter the circuit's behavior at $\omega_o = 1\,M\,r/s$? Because the L and C act like a short circuit at ω_o, R_2 is bypassed and leaves the circuit response unaltered at ω_o.

Note: Finding the cutoff frequencies for this circuit is difficult owing to the R's from the L and C in the expression for $H(j\omega)$.