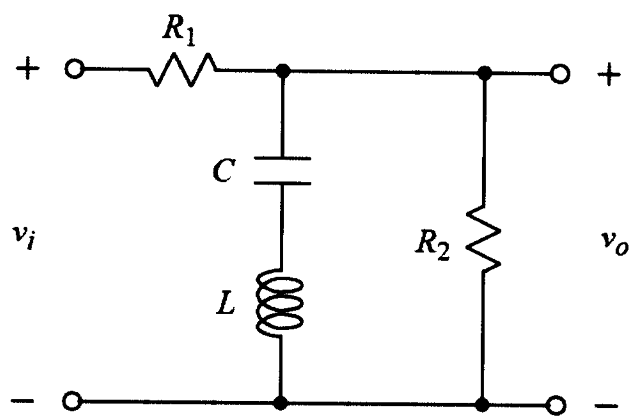
HOMEWORK #3 prob 5 solution

Ex:



$R_1 = 18\ \Omega$ $\quad R_2 = 36\ \Omega$ $\quad C = 31.25\ \mu\text{F}$ $\quad L = 2\text{ mH}$

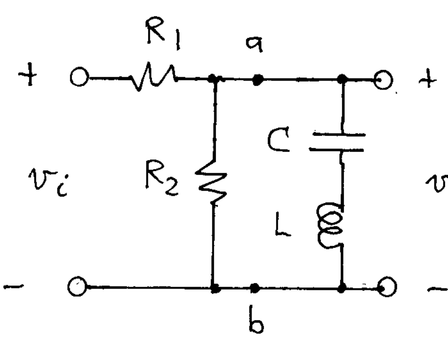
a) For the band-reject filter shown above, determine the transfer function V_o/V_i.
Hint: move R_2 to the left of L and C, and use a Thevenin equivalent.

b) Find ω_o

c) Find ω_{C1} and ω_{C2}

d) Find β and Q

sol'n: a) We redraw the circuit with R_2 to the left of L and C:



We remove L and C to find the Thevenin equivalent of v_i, R_1, and R_2. We then measure the voltage across R_2. The voltage-divider formula yields the answer:

b) $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2m \cdot 31.25\mu}}$ r/s

$\omega_o = 4$ k r/s

c) To find ω_{c1} and ω_{c2}, we solve

$$|H| = \frac{1}{\sqrt{2}} \max_{\omega} |H|$$

The max $|H|$ will occur when the imaginary part in the denominator is zero. This occurs when $\omega = 0$ or $\omega \to \infty$. The value we get is

$$\max_{\omega} |H| = \frac{R_2}{R_1 + R_2}$$

So we solve

$$|H| = \left|\cancel{\frac{R_2}{R_1 + R_2}}\right| \left| \frac{1}{1 - j\frac{R_{Th}}{\omega L - \frac{1}{\omega C}}} \right| = \cancel{\frac{R_1}{R_1 + R_2}} \cdot \frac{1}{\sqrt{2}}$$

The sol'n will occur when

$$\left| 1 - j\frac{R_{Th}}{\omega L - \frac{1}{\omega C}} \right| = \sqrt{2}$$

Since $|a + jb| = \sqrt{a^2 + b^2}$

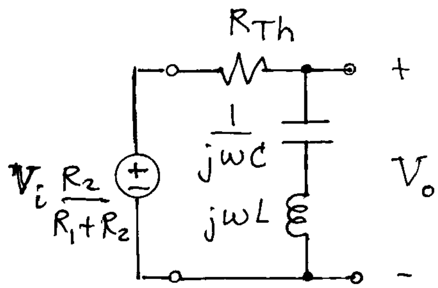
we must have $\frac{R_{Th}}{\omega L - \frac{1}{\omega C}} = \pm 1.$

$$v_{Th} = v_i \frac{R_2}{R_1 + R_2}$$

To find R_{Th}, we turn off v_i, which becomes a wire, and look in from a, b. We see $R_{Th} = R_1 \| R_2$.

$$R_{Th} = R_1 \| R_2$$

Our new circuit model: (freq. domain)



Our transfer function is

$$H(j\omega) = \frac{V_o}{V_i} = \frac{\dfrac{\dfrac{1}{j\omega C} + j\omega L}{R_{Th} + \dfrac{1}{j\omega C} + j\omega L} \cdot \dfrac{V_i R_2}{R_1 + R_2}}{V_i}$$

$$= \frac{R_2}{R_1 + R_2} \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R_{Th} + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Dividing top and bottom by $j\left(\omega L - \frac{1}{\omega C}\right)$ yields a cleaner form:

$$H(j\omega) = \frac{R_2}{R_1 + R_2} \frac{1}{1 - j \dfrac{R_{Th}}{\omega L - \frac{1}{\omega C}}} = \frac{2/3}{1 - j \dfrac{12}{\omega \cdot 2m - \frac{1}{3125\mu\omega}}}$$

or $R_{Th} = \pm\left(\omega L - \frac{1}{\omega C}\right)$

or $\pm R_{Th} = \omega L - \frac{1}{\omega C}$

or $\pm R_{Th}\,\omega = \omega^2 L - \frac{1}{C}$

or $\omega^2 L \pm R_{Th}\,\omega - \frac{1}{C} = 0$

or $\omega^2 \pm \frac{R_{Th}}{L}\,\omega - \frac{1}{LC} = 0$

or $\omega_{c1,2} = \pm \frac{R_{Th}}{2L} \pm \sqrt{\left(\frac{R_{Th}}{2L}\right)^2 + \frac{1}{LC}}$

We use the $\omega > 0$ roots:

$$\omega_{c1,2} = \pm \frac{R_{Th}}{2L} + \sqrt{\left(\frac{R_{Th}}{2L}\right)^2 + \frac{1}{LC}}$$

$$\frac{R_{Th}}{2L} = \frac{12\,\Omega}{2(2\,mH)} = 3k\ r/s$$

$$\frac{1}{LC} = (4k)^2 (r/s)^2 = \omega_o^2 \text{ (above)}$$

$$\omega_{c1,2} = \pm 3k + \sqrt{(3k)^2 + (4k)^2}\ r/s = \pm 3k + 5k\ r/s$$

$$\omega_{c1,2} = 2k\ r/s \text{ and } 8k\ r/s$$

d) $\beta \equiv \omega_{c2} - \omega_{c1} = \frac{R_{Th}}{L} = \frac{12\,\Omega}{2\,mH} = 6k\ r/s$

$$Q \equiv \frac{\omega_o}{\beta} = \frac{4k\ r/s}{6k\ r/s} = \frac{2}{3}$$