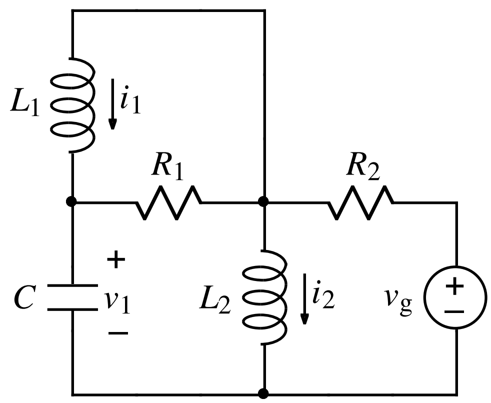
HOMEWORK #2 prob 3 solution

Ex:



At $t = 0$, $v_g(t)$ switches instantly from $-v_o$ to v_o.

a) Write the state-variable equations for the circuit in terms of the state vector:

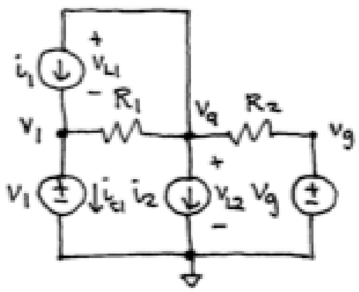
$$\vec{x} = \begin{bmatrix} i_1 \\ i_2 \\ v_1 \end{bmatrix}$$

b) Evaluate the state vector at $t = 0^+$.

SOL'N: a)

- Write first derivatives of state variables on the left side of eq'ns.
- Use $\frac{di_L}{dt} = \frac{v_L}{L}$ and $\frac{dv_C}{dt} = \frac{i_C}{C}$ to get non-derivatives on the right of each eq'n.
- Write v_L or i_C as a function of only state variables (no derivatives), i_L's and v_C's, and component or source values.

It is visually helpful to draw L's and C's as i_L sources and v_C sources. Then we use Kirchhoff's Laws and Ohm's Law or more formal methods such as node-v to write expressions for v_L's and i_C's.



Here, the circuit is complicated enough to warrant the use of node-v method.

v_a node: $$\frac{v_a - v_1}{R_1} + i_1 + i_2 + \frac{v_a - v_g}{R_2} = 0\,A$$

$$v_a\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v_1}{R_1} + \frac{v_g}{R_2} - i_1 - i_2$$

mult by $R_1 R_2$ $$v_a(R_2 + R_1) = R_2 v_1 + v_g R_1 - (i_1 + i_2)\cdot R_1 R_2$$

$$v_a = \frac{R_2 v_1 + v_g R_1 - (i_1 + i_2) R_1 R_2}{R_1 + R_2}$$

We can now use the expression for v_a:

$$v_{L1} = v_a - v_1$$

$$v_{L2} = v_a$$

$$i_{C1} = i_1 + \frac{v_a - v_1}{R_1}$$

Now we substitute for v_a and the above eq'ns in the state eq'ns: (Note: $V_g = +v_o$)

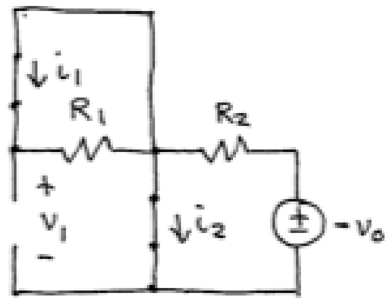
$$\frac{di_{L1}}{dt} = \frac{di_1}{dt} = \frac{1}{L_1}\left(\frac{R_2 v_1 + v_g R_1 - (i_1 + i_2) R_1 R_2}{R_1 + R_2} - v_1\right)$$

$$\frac{di_{L2}}{dt} = \frac{di_2}{dt} = \frac{1}{L_2}\left(\frac{R_2 v_1 + v_g R_1 - (i_1 + i_2) R_1 R_2}{R_1 + R_2}\right)$$

$$\frac{dv_C}{dt} = \frac{dv_1}{dt} = \frac{1}{C}\left(i_1 + \frac{R_2 v_1 + v_g R_1 - (i_1 + i_2) R_1 R_2}{R_1 (R_1 + R_2)} - \frac{v_1}{R_1}\right)$$

b) For the state vector at $t=0^+$, we find the state vector at $t=0^-$. (The state vars are energy vars that can't change instantly.)

$t=0^-$ model: C = open, L = wire, $v_g = -V_0$



$i_1(0^-) = 0$ since there is open circuit at v_1 and R_1 is shorted, (so no current in any direction from node on left side of R_1).

$i_2(0^-) = -\frac{V_0}{R_2}$ from loop on right side

$v_1(0^-) = 0$ since short at i_2 and short across R_1 leaves 0V for v_1 in v-loop lower left.

$$\begin{bmatrix} i_1(0^+) \\ i_2(0^+) \\ v_1(0^+) \end{bmatrix} = \begin{bmatrix} 0\text{ A} \\ -V_0/R_2 \\ 0\text{ V} \end{bmatrix}$$