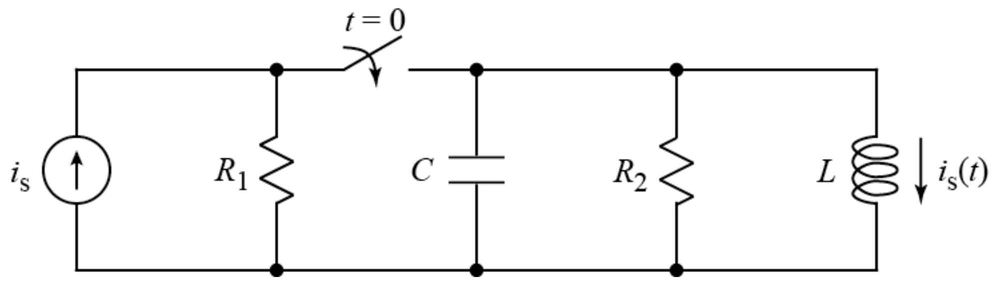
Ex:



After being open for a long time, the switch closes at $t = 0$.

$i_s = 4$ mA $C = 2$ μF $R_1 = 200$ Ω $R_2 = 200$ Ω

a) If $L = 125$ mH, find the characteristic roots, s_1 and s_2, for the above circuit.

b) If $L = 11.834$ mH, find the damping frequency, ω_d.

c) Find the value of L that makes the circuit critically-damped, and find $i_L(t)$ for that value of L.

SOL'N: a) Because the switch closes at $t = 0$, all of the components are in the circuit that we use to calculate the characteristic roots. All the components are in parallel, so we have a parallel RLC circuit, with the two R's in parallel acting as the following equivalent R:

$$R = R_1 \parallel R_2 = 200\,\Omega \parallel 200\,\Omega = 100\,\Omega$$

For a parallel RLC circuit, α is found as

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 100\Omega \cdot 2\mu\text{F}} = 2.5\text{ k/s}$$

The resonant frequency squared is always $1/LC$:

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{125\,\text{mH} \cdot 2\mu\text{F}} = (2\,\text{kr/s})^2$$

The formula for the characteristic roots is always as follows:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

Using our circuit, we obtain the roots:

$$s_{1,2} = -2.5\text{k} \pm \sqrt{(2.5\text{k})^2 - (2\text{k})^2}\text{ r/s} = -2.5\text{k} \pm 1.5\text{k r/s}$$

or

$$s_{1,2} = -1\text{kr/s and } -4\text{kr/s}$$

SOL'N: b) Repeating the calculations from part (a) with the new value of L yields the following results, (α is unchanged):

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{11.834\,\text{mH}\cdot 2\mu\text{F}} \approx (6\text{kr/s})^2$$

$$s_{1,2} = -2.5\text{k} \pm \sqrt{(2.5\text{k})^2 - (6.5\text{k})^2}\ \text{r/s} \approx -2.5\text{k} \pm j6\text{k r/s}$$

The damping frequency is the imaginary part of the roots, which may also be found by taking the negative of the quantity under the square root (so as to obtain a positive value):

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} \approx \sqrt{(6.5\text{k})^2 - (2.5\text{k})^2} = 6\text{ kr/s}$$

SOL'N: c) Critical damping occurs when $\alpha = \omega_o$. Repeating the calculations from part (a) with the new value of L yields the following results, (α is unchanged):

$$\alpha = \frac{1}{2RC} = \omega_o = \frac{1}{\sqrt{LC}}$$

If we invert both sides of the equation, we have the following result:

$$2RC = \sqrt{LC}$$

Using this equation, we solve for L:

$$L = 4R^2C = 4(100\Omega)^2 2\mu\text{F} = 80\,\text{mH}$$

The value of α was found in part (a) and is minus the value of the repeated root:

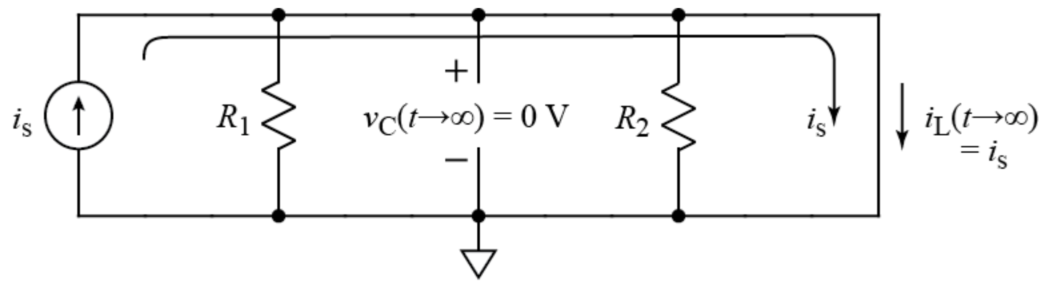
$$s_{1,2} = -\alpha = -2.5\,\text{kr/s}$$

For critical damping, we use the following form of solution:

$$i_L(t) = A_1e^{-\alpha t} + A_2te^{-\alpha t} + A_3$$

Now we start the circuit analysis.

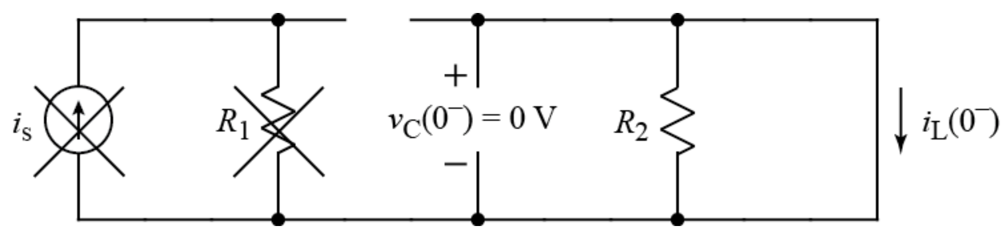
t->∞: (A_3 = final value)



All of i_s will flow through the short formed by the L as time approaches infinity, so the final value of i_L and the value of A_3 will be zero.

$$A_3 = i_s = 4\,\text{mA}$$

t = 0⁻: (initial conditions on L and C)



The initial conditions on the L and C will be zero, since there is no source in the circuit for $t < 0$. We match this to the critically damped symbolic solution at $t = 0^+$:

$$i_L(0^+) = 0\,\text{A} = A_1 + A_3 = A_1 + 4\,\text{mA}$$

or

$$A_1 = -4\,\text{mA}$$

Now we match the derivative of the inductor current found from the circuit to the symbolic solution at $t = 0^+$:

$$\left.\frac{di_L(t)}{dt}\right|_{t=0^+} = -\alpha A_1 + A_2 = \frac{v_L(0^+)}{L} = \frac{v_C(0^+)}{L} = 0\,\text{A/s}$$

or

$$A_2 = \alpha A_1 = 2.5\text{k}(-4\,\text{m})\,\text{A/s} = -10\,\text{A/s}$$

Thus, $i_L(t) = -4\text{mA}\cdot e^{-2.5\text{k}t} - 10\text{A}te^{-2.5\text{k}t} + 4\text{mA}$.