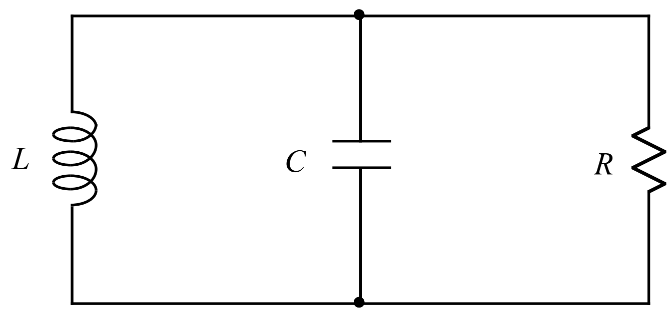
Ex:



$L = 2.5$ nH $\qquad C = 1.6$ nF $\qquad R = 0.625\ \Omega$

a) Find the characteristic roots, s_1 and s_2, for the above circuit.

b) Is the circuit over-damped, critically-damped, or under-damped? Explain your answer.

c) If the L and C values in the circuit are increased by a factor of 10, (and R remains the same), will the circuit be over-damped, critically-damped, or under-damped? Justify your answer with calculations.

SOL'N: a) For a parallel RLC circuit (or a series RLC circuit), we have the following formula for the characteristic roots:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

For a parallel RLC circuit, the value of α is one-half the inverse RC time constant:

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 0.625\Omega \cdot 1.6nF} = \frac{1\text{ G/s}}{2} = 500\text{ M/s}$$

For both parallel and series RLC circuits, the resonant frequency, ω_o, is the inverse of the square root of the product of L and C:

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{2.5\text{ nH} \cdot 1.6\text{ nF}} = \left(\frac{1}{2\text{ n}}\text{ r/s}\right)^2 = (500\text{ Mr/s})^2$$

We find that, since $\alpha = \omega_o$, the roots are equal:

$$s_{1,2} = -500\text{ Mr/s} \pm \sqrt{(500\text{ kr/s})^2 - (500\text{ kr/s})^2} = -500\text{ Mr/s}$$

SOL'N: b) When the two roots are the same, the circuit is critically-damped.

SOL'N: c) If both L and C increase by a factor of 10, the resonant frequency, ω_o, will decrease by a factor of $\sqrt{10} \cdot \sqrt{10} = 10$.

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_o = \frac{1}{\sqrt{LC}} = \frac{500 \text{ Mr/s}}{\sqrt{10}\sqrt{10}} = 50 \text{ Mr/s}$$

Because C increases by a factor of 10, the value of α will also decrease by a factor of 10:

$$\alpha = \frac{500 \text{ Mr/s}}{10} 50 \text{ Mr/s}$$

Thus, it is still the case that $\alpha = \omega_o$, the two roots are the same, and the circuit is critically-damped.