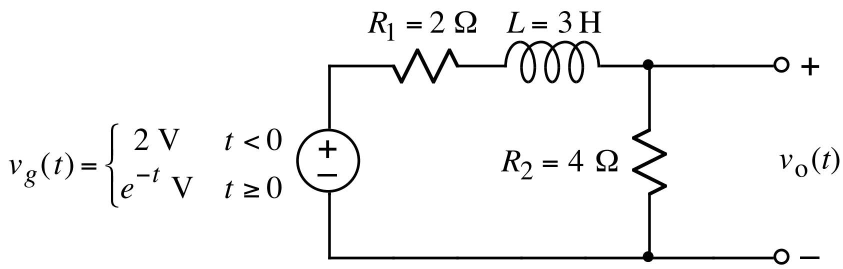
ECE 2260
F 09

HOMEWORK #6 probs 4 and 5 solutions

Ex:



a) Write the Laplace transform $V_g(s)$ of $v_g(t)$.

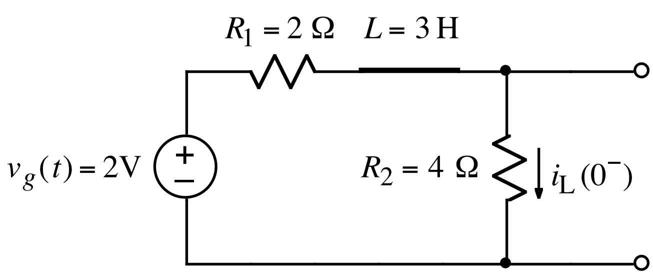
b) Write the Laplace transform $V_o(s)$ of $v_o(t)$. Be sure to include the effects of initial conditions, if they are nonzero.

c) Write a numerical time-domain expression for $v_o(t)$ where $t \geq 0$.

SOL'N: a) The Laplace transform depends only on what the input signal is from time 0 to ∞.

$$\mathcal{L}\{v_g(t)\} = \mathcal{L}\{e^{-t}\}\mathrm{V} = \frac{1}{s+1}\mathrm{V}$$

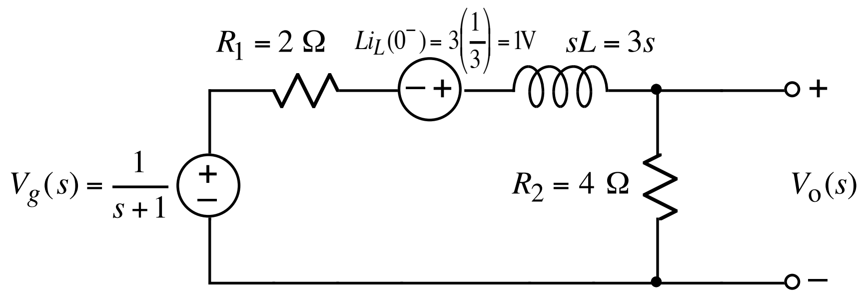
b) We find initial conditions by considering circuit at $t = 0^-$. The circuit has a 2 V input and has reached equilibrium. Thus, the L acts like a wire.



We use Ohm's Law to obtain the initial inductor current:

$$i_L(0^-) = \frac{2\ \mathrm{V}}{2\ \Omega + 4\ \Omega} = \frac{1}{3}\ \mathrm{A}$$

Our circuit model includes initial conditions for the L:



The output voltage is found by summing the input voltages and using a voltage divider formula:

$$V_o(s) = \left[\frac{1}{s+1} + Li_L(0^-)\right]\frac{R_2}{R_1 + sL + R_2} = \left[\frac{1}{s+1} + 3\left(\frac{1}{3}\right)\right]\frac{4}{2+3s+4}$$

$$= \left[\frac{1}{s+1} + 1\right]\frac{4}{3(s+2)} = \frac{4}{3}\frac{s+2}{(s+1)(s+2)}$$

$$= \frac{4}{3}\frac{1}{s+1}\ \text{V}$$

c) The output voltage versus time is the inverse Laplace transform of $V_o(t)$:

$$v_o(t \geq 0) = \frac{4}{3}e^{-t}u(t)\ \text{V}$$

NOTE: We could omit the $u(t)$, but it reminds us that our answer only applies to $t \geq 0$.