

Ex: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{s^2 + 11s + 37}{s^2 + 8s + 25}$$

SOL'N: $F(s)$ is an improper fraction, so our first step is to write $F(s)$ as an integer plus a proper fraction.

$$F(s) = \frac{s^2 + 11s + 37}{(s+4)^2 + 3^2} = \frac{(s+4)^2 + 3^2}{(s+4)^2 + 3^2} + \frac{3s+12}{(s+4)^2 + 3^2} = 1 + \frac{3s+12}{(s+4)^2 + 3^2}$$

Now we note that the inverse transform of 1 is $\delta(t)$ and consider only the proper fraction from now on.

When we factor the denominator to find the partial fraction terms needed, we discover that we have complex roots. Consequently, we write terms that correspond to a decaying cosine and decaying sine. We identify $a = 4$ and $\omega = 3$ as the decay rate and frequency of oscillation.

$$F(s) = \frac{3s+12}{(s+4)^2 + 3^2} = \frac{A(s+4)}{(s+4)^2 + 3^2} + \frac{B(3)}{(s+4)^2 + 3^2}$$

We find A and B by making the numerators the same on left and right.

$$3s+12 = A(s+4) + B(3)$$

Starting with the highest power of s , we find the value of A and B . Since the only term multiplying s on the right is A , we must have the following result:

$$A = 3$$

Using this value for A , we can quickly find B .

$$3s+12 = 3(s+4) + B(3) = 3s+12 + B(3)$$

or

$$B = 0$$

Now we take the inverse transform to obtain our final result.

$$f(t) = \delta(t) + 3e^{-4t} \cos(3t)u(t)$$

NOTE: We multiply by $u(t)$ as a reminder that we are uncertain of the value of $f(t)$ for $t < 0$. Since $\delta(t) = 0$ for $t > 0$, we need only multiply the second term by $u(t)$.