HOMEWORK #5 prob 3a solution

Ex: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{-6s^3 - 19s^2 + 42s + 72}{s^4 + 7s^3 + 12s^2}$$

SOL'N: Factor the denominator to find the partial fraction terms needed.

$$F(s) = \frac{-6s^3 - 19s^2 + 42s + 72}{s^2(s+3)(s+4)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+3} + \frac{D}{s+4}$$

we can find all but B by the pole cover-up method.

$$A = s^2 F(s)\Big|_{s=0} = \frac{72}{3 \cdot 4} = 6$$

$$C = (s+3)F(s)\Big|_{s=-3} = \frac{-6(-3)^3 - 19(-3)^2 + 42(-3) + 72}{(-3)^2(-3+4)}$$

$$= \frac{[-6(-3) - 19](-3)^2 - 54}{9} = \frac{-63}{9} = -7$$

$$D = (s+4)F(s)\Big|_{s=-4} = \frac{-6(-4)^3 - 19(-4)^2 + 42(-4) + 72}{(-4)^2(-4+3)}$$

$$= \frac{[-6(-4) - 19](-4)^2 - 96}{-16} = \frac{-16}{-16} = 1$$

An easy way to find B is to choose a convenient value for s and equate $F(s)$ with the partial fraction expression. (This works because the only coefficient left to be found is B.) Here, we use $s = 1$.

$$F(s=1) = \frac{-6 - 19 + 42 + 72}{1^2(1+3)(1+4)} = \frac{6}{1^2} + \frac{B}{1} + \frac{-7}{1+3} + \frac{1}{1+4}$$

or

$$F(s=1) = \frac{89}{20} = 6 + \frac{B}{1} + \frac{-7}{4} + \frac{1}{5} = B + \frac{120 - 7(5) + 4}{20} = B + \frac{89}{20}$$

or

$$B = 0$$

To check our answer, we convert the partial fraction expression into a ratio of polynomials and verify that we get $F(s)$.

$$\frac{6}{s^2}+\frac{0}{s}+\frac{-7}{s+3}+\frac{1}{s+4}$$
$$=\frac{6(s+3)(s+4)+0-7s^2(s+4)+s^2(s+3)}{s^2(s+3)(s+4)}$$
$$=\frac{-6s^3-19s^2+42s+72}{s^4+7s^3+12s^2}$$
$$=F(s) \quad \text{(result is verified } \surd\text{)}$$

Now we take the inverse transform of the partial fraction terms to get our final result.

$$f(t)=[6t-7e^{-3t}+e^{-4t}]u(t)$$

NOTE: We multiply by $u(t)$ as a reminder that we are uncertain of the value of $f(t)$ for $t < 0$.