

**Ex:** Find the Laplace transform of the following waveform:

$$t^2 \frac{d}{dt} [e^{-4t} \cos(3t)]$$

**SOL'N:** We start on the inside (of this layered onion) and apply identities to work our way out to the time-domain form given. The innermost term, found in a transform table, is  $e^{-4t} \cos(3t)$ .

$$\mathcal{L}\{e^{-4t} \cos(3t)\} = \frac{s+4}{(s+4)^2 + 3^2}$$

Now we apply the identity for derivatives.

$$\mathcal{L}\left\{\frac{d}{dt} f(t)\right\} = sF(s) - f(0^-)$$

or

$$\mathcal{L}\left\{\frac{d}{dt} e^{-4t} \cos(3t)\right\} = s \frac{s+4}{(s+4)^2 + 3^2} - e^{-4t} \cos(3t)|_{t=0^-}$$

or

$$\mathcal{L}\left\{\frac{d}{dt} e^{-4t} \cos(3t)\right\} = \frac{s(s+4)}{(s+4)^2 + 3^2} - 1$$

Now we apply the identity for multiplication by  $t^n$ .

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n F(s)}{ds^n}$$

or

$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = (-1)^2 \frac{d^2}{ds^2} \left[ \frac{s(s+4)}{(s+4)^2 + 3^2} - 1 \right]$$

or

$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = (-1)^2 \frac{d^2}{ds^2} \left[ \frac{s(s+4)}{(s+4)^2 + 3^2} - \frac{(s+4)^2 + 3^2}{(s+4)^2 + 3^2} \right]$$

or

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$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = \frac{d^2}{ds^2} \left[ -\frac{4s+25}{(s+4)^2 + 3^2} \right]$$

or

$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = -\frac{d}{ds} \left[ \frac{4}{(s+4)^2 + 3^2} - \frac{4s+25}{[(s+4)^2 + 3^2]^2} \cdot (2s+8) \right]$$

or

$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = -\frac{d}{ds} \left[ \frac{4[(s+4)^2 + 3^2] - [4s+25][2s+8]}{[(s+4)^2 + 3^2]^2} \right]$$

or

$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = -\frac{d}{ds} \left[ \frac{-4s^2 - 50s - 100}{[(s+4)^2 + 3^2]^2} \right]$$

or

$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = -\left[ \frac{-8s-50}{[(s+4)^2 + 3^2]^2} - 2 \frac{-4s^2 - 50s - 100}{[(s+4)^2 + 3^2]^3} (2s+8) \right]$$

or, (with assistance from Matlab<sup>®</sup> symbolic computations),

$$\mathcal{L}\left\{t^2 \frac{d}{dt} e^{-4t} \cos(3t)\right\} = -2 \left[ \frac{4s^3 + 75s^2 + 300s + 175}{[(s+4)^2 + 3^2]^3} \right]$$