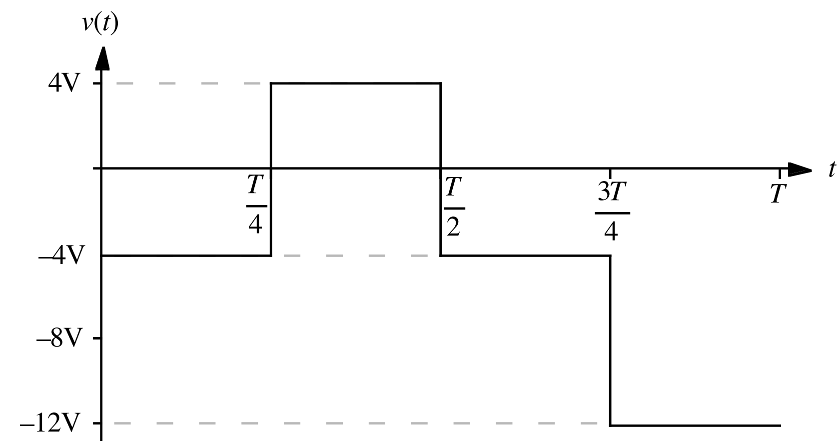
HOMEWORK #4 probs 2 and 3 solution

Ex:



One period, T, of a function $v(t)$ is shown above. The formula for $v(t)$ is

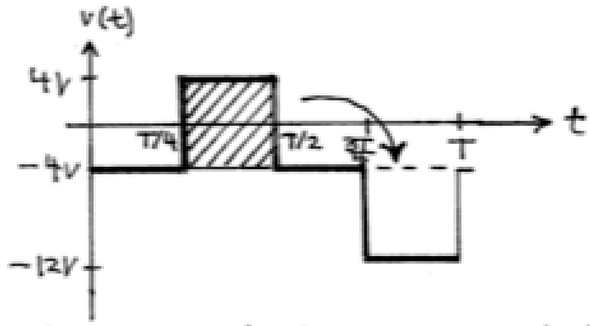
$$v(t) = \begin{cases} -4\text{ V} & 0 < t < T/4 \\ 4\text{ V} & T/4 < t < T/2 \\ -4\text{ V} & T/2 < t < 3T/4 \\ -12\text{ V} & 3T/4 < t < T \end{cases}$$

Find the numerical value of the following coefficients of the Fourier series for $v(t)$: (Hint: remove the DC offset after finding its value.)

a) a_v b) a_1

Ex: Find the value of b_1 and b_2 for the Fourier series.

soln: a) By inspection, the average value of the waveform is −4V. One way to see this is to imagine moving the block from T/4 to T/2 above −4V to 3T/4 to T:



If we wish to use an integral, we have the following:

$$a_v = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\left(\int_0^{T/4} -4V\,dt + \int_{T/4}^{T/2} 4V\,dt + \int_{T/2}^{3T/4} -4V\,dt + \int_{3T/4}^{T} -12V\,dt\right)$$

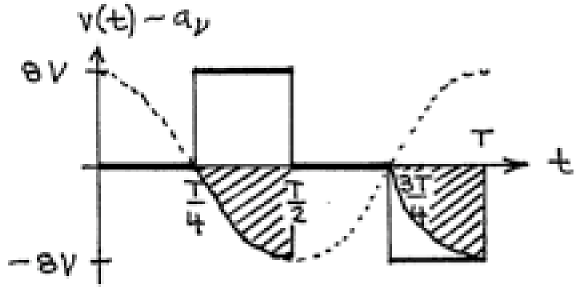
$$= \frac{1}{T}\left(-4Vt\Big|_0^{\frac{T}{4}} + 4Vt\Big|_{T/4}^{\frac{T}{2}} + -4Vt\Big|_{T/2}^{\frac{3T}{4}} + -12Vt\Big|_{3T/4}^{T}\right)$$

$$= \frac{1}{\cancel{T}}\left(\cancel{-4V\cdot\frac{T}{4}} \cancel{+4V\frac{T}{4}} - 4V\frac{\cancel{T}}{4} - 12V\frac{\cancel{T}}{4}\right)$$

$$= -\frac{16}{4}V$$

$$a_v = -4V$$

Another approach would be to remove the DC offset from $v(t)$ before calculating a_1.



We double the area from $\frac{T}{4}$ to $\frac{T}{2}$:

$$a_1 = 2 \cdot \frac{2}{T} \int_{T/4}^{T/2} 8V \cos\left(\frac{2\pi}{T} t\right) dt$$

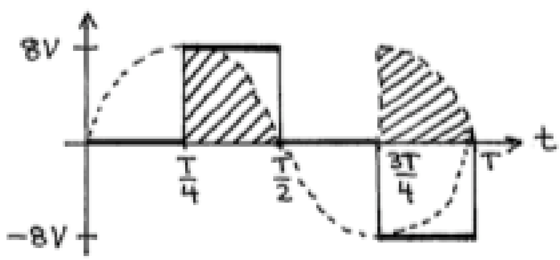
$$= \frac{4}{T} \cdot 8V \left. \frac{\sin\left(\frac{2\pi}{T} t\right)}{\frac{2\pi}{T}} \right|_{T/4}^{T/2}$$

$$= \frac{16V}{\pi} \Big[\underbrace{\sin(\pi)}_{=0} - \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1} \Big]$$

$$a_1 = -\frac{16}{\pi} V$$

$$b_1 = \frac{2}{T}\int_0^T v(t)\sin(\omega_0 t)\,dt$$

We sketch $[v(t) - a_v] \sin(\omega_0 t)$. That is, we remove the DC offset of $v(t)$.



We double the area from $\frac{T}{4}$ to $\frac{T}{2}$:

$$b_1 = 2 \cdot \frac{2}{T}\int_{T/4}^{T/2} 8V \sin\left(\frac{2\pi t}{T}\right) dt$$

$$= \frac{32V}{T}\left.\frac{\left[-\cos\left(\frac{2\pi t}{T}\right)\right]}{\frac{2\pi}{T}}\right|_{T/4}^{T/2}$$

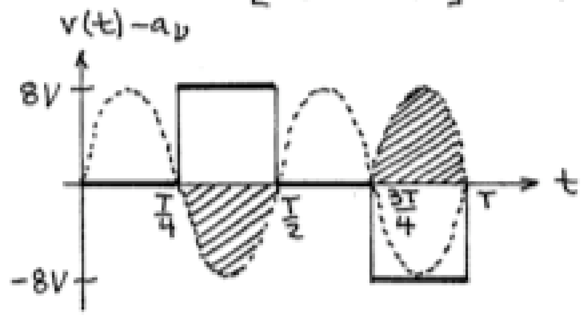
$$= \frac{16V}{\pi}(- -1 - -0)$$

$$= \frac{16}{\pi}V$$

We could also have observed that the areas are just negatives of the areas sketched for the calculation of b_1. Thus, $b_1 = -a_1 = \frac{16V}{\pi}$.

$$b_2 = \frac{2}{T}\int_0^T v(t)\sin(2\omega_0 t)\,dt$$

We sketch $[v(t) - a_v]\sin(2\omega_0 t)$.



The areas cancel. Thus,

$$b_2 = 0\ V$$