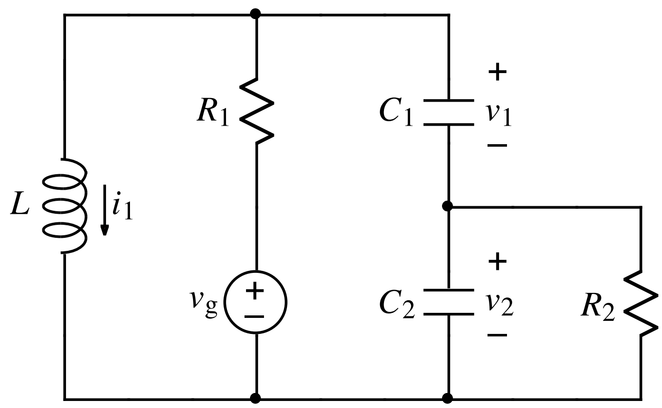
HOMEWORK #2 prob 3 solution

Ex:



At $t = 0$, $v_g(t)$ switches instantly from $-v_o$ to v_o.

a) Write the state-variable equations for the circuit in terms of the state vector:

$$\vec{x} = \begin{bmatrix} i_1 \\ v_1 \\ v_2 \end{bmatrix}$$

b) Evaluate the state vector at $t = 0^+$.

SOL'N:

a) We have first derivatives of state vars on the left, and we first equate these to non-derivatives:

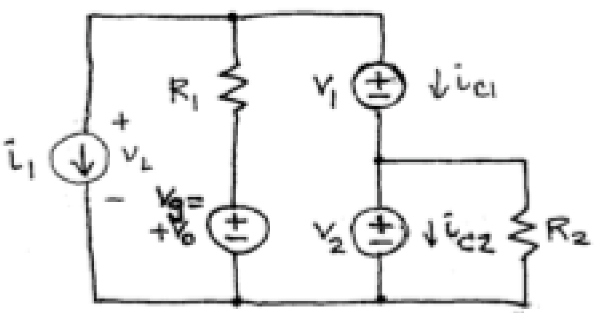
$$\frac{di_1}{dt} = \frac{di_L}{dt} = \frac{v_L}{L}$$

$$\frac{dv_1}{dt} = \frac{dv_{C1}}{dt} = \frac{i_{C1}}{C_1}$$

$$\frac{dv_2}{dt} = \frac{dv_{C2}}{dt} = \frac{i_{C2}}{C_2}$$

We now write the variables on the right in terms of only state var's i_1, v_2, and v_3, (and components and sources).

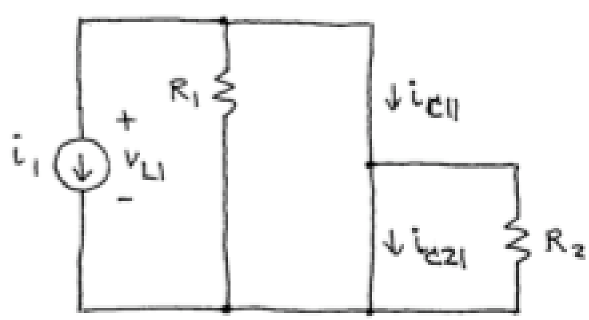
This problem of writing i's and v's in terms of state var's is easier to solve if we redraw the circuit with i_L's and v_C's shown as sources:



Note: we use $V_g = +V_0$ for $t > 0$.

Using superposition, we turn on one source at a time.

I. i_1 on

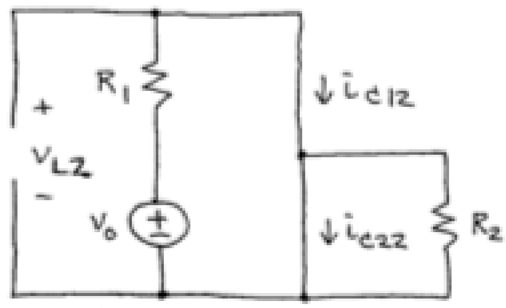


$v_{L1} = 0V$ (C_1, C_2 form wire, short)

$i_{C1} = -i_1$ (all of i_1 flows in short)

$i_{C2} = -i_1$ (all of i_1 flows in short)

II. $v_g = v_o$ on

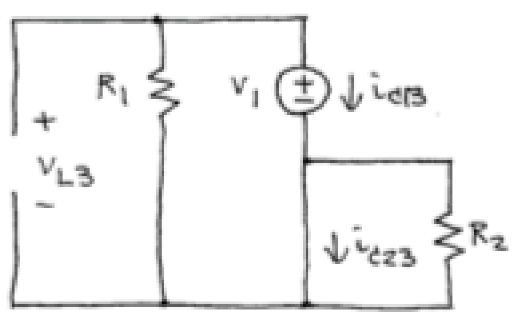


$v_{L2} = 0V$ (shorted by C_1, C_2

$i_{c12} = \dfrac{v_o}{R_1}$ (middle loop carries all of the current)

$i_{c22} = \dfrac{v_o}{R_1}$ (ditto, R_2 shorted)

III. v_1 on

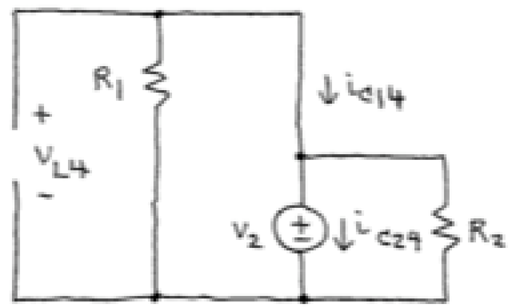


$v_{L3} = v_1$ (L is connected across v_1)

$i_{c13} = -\dfrac{v_1}{R_1}$ (all current is in middle loop)

$i_{c23} = i_{c13} = -\dfrac{v_1}{R_1}$

IV. v_2 on



$v_{L4} = v_2$ (L is across v_2)

$i_{c14} = \dfrac{-v_2}{R_1}$ (i flows around middle loop)

$i_{c24} = -\dfrac{v_2}{R_1 \| R_2} = -\dfrac{v_2}{R_1} - \dfrac{v_2}{R_2}$ (i in R_1 & R_2 from Ohm's law)

Combining results:

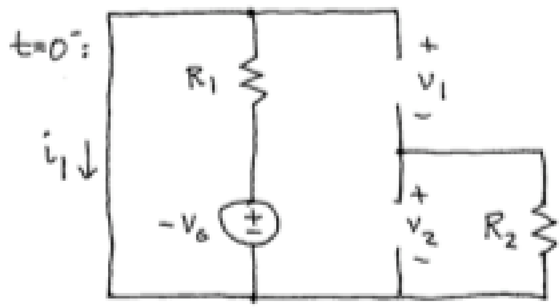
$$\frac{di_1}{dt} = \frac{v_1 + v_2}{L}$$

$$\frac{dv_1}{dt} = \frac{-i_1 - \frac{v_1}{R_1} - \frac{v_2}{R_1} + \frac{v_0}{R_1}}{C_1}$$

$$\frac{dv_2}{dt} = \frac{-i_1 - \frac{v_1}{R_1} - \frac{v_2}{R_1} - \frac{v_2}{R_2} + \frac{v_0}{R_1}}{C_2}$$

b)

For initial conditions, we use the circuit at $t=0^-$, since $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$. At $t=0^-$, L = wire, C = open, and $v_g = -V_0$.



From loop on left, $i_1(0^-) = -\frac{v_0}{R_1}$.

Note: no current flows on right side.

No current R_2, so $v_2(0^-) = 0V$ is v-drop across R_2.

Short on left side and no v-drop for R_2 give $v_1 = 0V$ from outer v-loop.

$$i_1(0^+) = -\frac{v_0}{R_1}$$ (same values as at $t=0^-$)

$$v_1(0^+) = 0V$$

$$v_2(0^+) = 0V$$