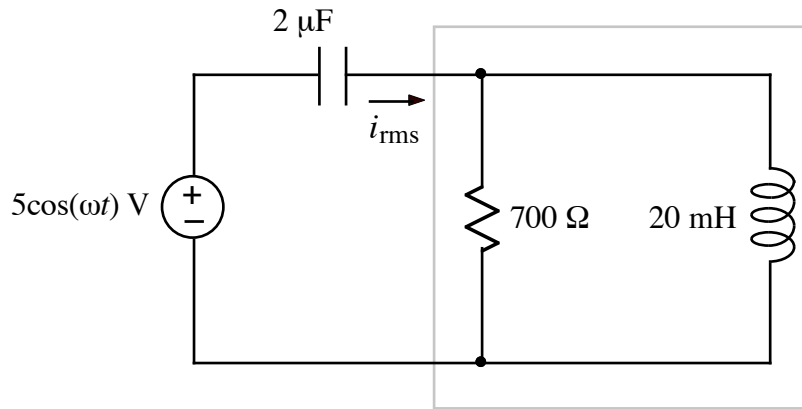


HW 7 prob 3 solution

Ex:



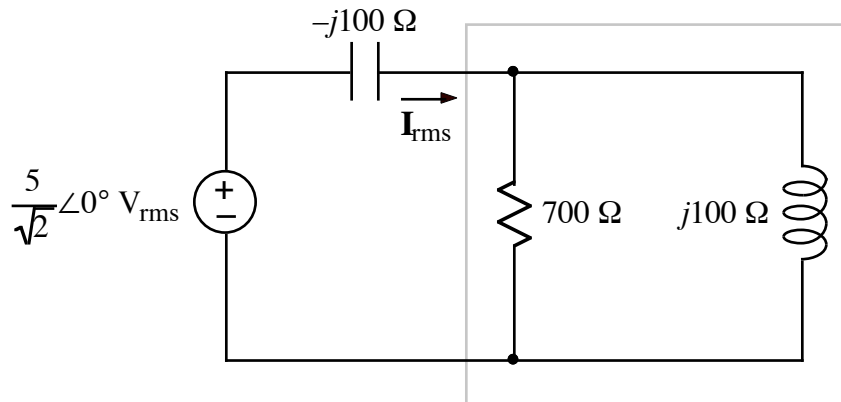
- a) Calculate I_{rms} . Note: $\omega = 5 \text{ kr/s}$.
- b) Calculate the complex power, S , for the components inside the box.

SOL'N: a) We first create an s -domain model. We have the following impedance values:

$$\frac{1}{j\omega C} = \frac{1}{j5\text{kr/s} \cdot 2 \mu\text{F}} = -j100 \Omega$$

$$j\omega L = j5\text{kr/s} \cdot 20 \text{ mH} = j100 \Omega$$

Our s -domain model, with the rms value of the voltage source:



We use Ohm's law to calculate I_{rms} :

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{z_{\text{tot}}} = \frac{5/\sqrt{2} V_{\text{rms}}}{-j100 + 700 \parallel j100 \Omega} = \frac{5/\sqrt{2} V_{\text{rms}}}{100 \left(-j + \frac{j7}{7+j} \right) \Omega}$$

or

$$\mathbf{I}_{\text{rms}} = \frac{5/\sqrt{2} \text{ V}_{\text{rms}}}{100 \left(-j \frac{7+j}{7+j} + \frac{j7}{7+j} \right) \Omega} = \frac{5/\sqrt{2}}{100 \frac{1}{7+j}} \text{ A}_{\text{rms}}$$

or

$$\mathbf{I}_{\text{rms}} = \frac{50}{\sqrt{2}} (7+j) \text{ mA}_{\text{rms}} = \frac{50}{\sqrt{2}} \sqrt{50} \angle \tan^{-1} \left(\frac{1}{7} \right) \text{ mA}_{\text{rms}}$$

or

$$\mathbf{I}_{\text{rms}} = 50\sqrt{25} \angle 8.13^\circ \text{ mA}_{\text{rms}}$$

b) We use the rms formulas for S :

$$S = \mathbf{V}_{\text{rms}} \mathbf{I}_{\text{rms}}^* = \mathbf{I}_{\text{rms}} z \mathbf{I}_{\text{rms}}^* = |\mathbf{I}_{\text{rms}}|^2 z$$

Using numerical values, we have the following:

$$S = (50\sqrt{25} \text{ m})^2 \cdot 100 \cdot 7 \parallel j = \frac{25}{20^2} \cdot 100 \cdot \frac{j7}{7+j} \text{ VA}$$

or

$$S = \frac{25}{4} \cdot \frac{j7}{7+j} \cdot \frac{7-j}{7-j} = \frac{25}{4} \cdot \frac{7(1+j7)}{50} = \frac{7(1+j7)}{8} \text{ VA}$$

In polar form, we have the following:

$$S = \frac{7\sqrt{50}}{8} \tan^{-1} \left(\frac{7}{1} \right) = \frac{7\sqrt{50}}{8} \angle 81.7^\circ \text{ VA}$$

NOTE: Since S is a power quantity, there is no factor of $\sqrt{2}$ for rms after we have calculated S . A factor of $1/2$ has, however, been embedded in the final answer via the product of rms quantities.