HW #6 prob 1 solution

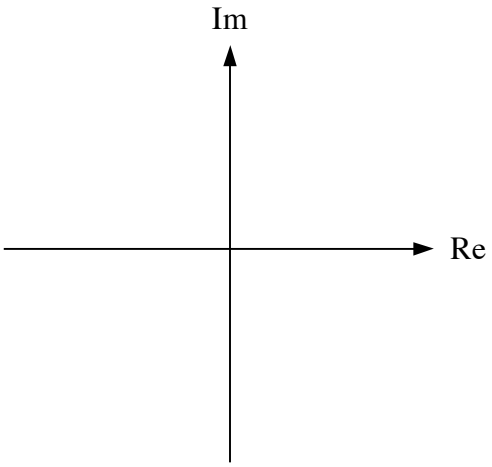
1. a) Find $\mathcal{L}\left\{\int_0^t e^{-6\tau}\cos(7\tau)d\tau\right\}$.

b) Find $v(t)$ if $V(s)=\dfrac{18s+148}{s^2+12s+11}$.

c) Find $\lim_{t\to\infty} v(t)$ if $V(s)=\dfrac{s^2+4}{(s+3)^3}$.

d) Plot the poles and zeros of $V(s)$ in the s plane.

$$V(s)=\frac{s^2+5s+6}{(s+1)\left[(s+4)^2+5^2\right]}$$



SOL'N: a) We use the identity for integration and the transform pair for the decaying cosine:

$$\mathcal{L}\left\{\int_0^t e^{-6\tau}\cos(7\tau)d\tau\right\}=\frac{1}{s}\mathcal{L}\left\{e^{-6t}\cos(7t)\right\}=\frac{s+6}{s[(s+6)^2+7^2]}$$

b) We have two real roots:

$$\mathcal{L}^{-1}\left\{\frac{18s+148}{s^2+12s+11}\right\}=\mathcal{L}^{-1}\left\{\frac{18s+148}{(s+1)(s+11)}\right\}=\mathcal{L}^{-1}\left\{\frac{A}{s+1}+\frac{B}{s+11}\right\}$$

We fine A and B by the usual method of removing the pole and evaluating at the pole value:

$$A=(s+1)V(s)\big|_{s=-1}=\left.\frac{18s+148}{s+11}\right|_{s=-1}=\frac{130}{10}=13$$

$$B = (s+11)V(s)\Big|_{s=-11} = \left.\frac{18s+148}{s+1}\right|_{s=-11} = \frac{-50}{-10} = 5$$

Before proceeding, we check our result:

$$\frac{13}{s+1} + \frac{5}{s+11} = \frac{13(s+11)+5(s+1)}{s^2+12s+11} = \frac{18s+143+5}{s^2+12s+11} \quad \surd$$

The inverse transform follows immediately:

$$\mathcal{L}^{-1}\left\{\frac{13}{s+1} + \frac{5}{s+11}\right\} = \left[13e^{-t} + 5e^{-11t}\right]u(t)$$

c) We use the final value theorem. Note that the theorem applies, since we have poles with negative real parts:

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} sV(s) = \lim_{s\to 0} s\frac{s^2+4}{(s+3)^3} = 0 \cdot \frac{4}{3^3} = 0$$

d) The zeros are the roots of the numerator. The poles are the roots of the denominator.

$$V(s) = \frac{s^2+5s+6}{(s+1)\left[(s+4)^2+5^2\right]} = \frac{(s+2)(s+3)}{(s+1)(s+4+j5)(s+4-j5)}$$

Zeros are plotted as o's at $s = -2$ and at $s = -3$.

Poles are plotted as x's at $s = -1$, $s = -4 - j5$, and at $s = -4 + j5$.