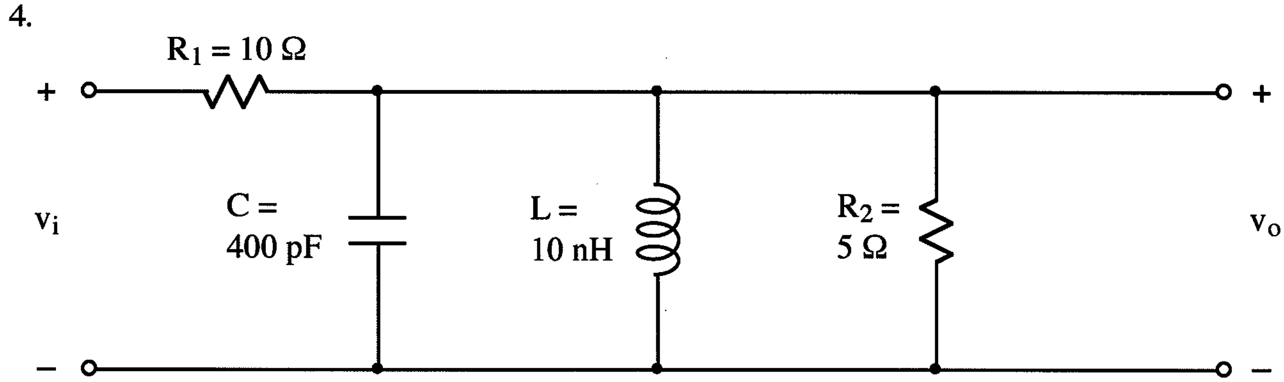
ECE 2260
F 08

HOMEWORK #3 prob 4 soln

4.



For the band-pass filter shown above, calculate the following quantities:
Hint: Use a Thevenin equivalent for the R's.

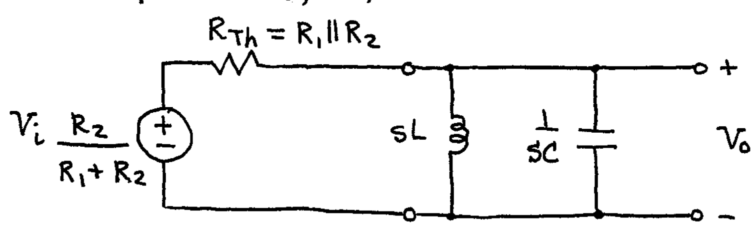
a) ω_o

b) ω_{C1} and ω_{C2}

c) β

d) Q

soln: a) We first move R_2 to the left side of L and C and replace v_i, R_1, and R_2 with a Thevenin equiv.



Now we have a standard RLC filter but with a scaling factor of $\dfrac{R_2}{R_1 + R_2}$. This scaling factor is real and scales the entire frequency response curve of $|H(s)|$ vs ω by the same factor. Thus $\omega_o^2 = \dfrac{1}{LC}$ still.

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{10n \cdot 400p} = \left(\frac{1}{2n}\right)^2$$

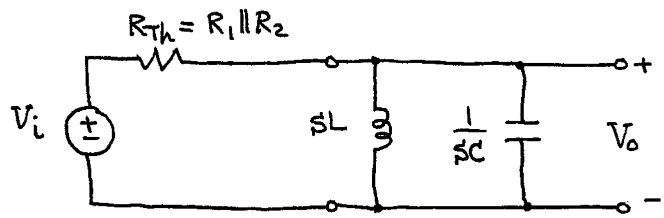
$$\omega_o = \frac{1}{2n} = 500 \text{ Mr/s}$$

b) To find ω_{c1}, ω_{c2} we solve $|H(s)| = \frac{1}{\sqrt{2}} \max_{\omega} |H(s)|$.

Because $\frac{R_2}{R_1+R_2}$ scales every frequency of $H(s)$ the same amount, we can ignore the $\frac{R_2}{R_1+R_2}$ and solve $|H'(s)| = \frac{1}{\sqrt{2}} \max_{\omega} |H'(s)| = \frac{1}{\sqrt{2}}$ (see below)

where $H'(s) = \dfrac{sL \| \frac{1}{sC}}{sL \| \frac{1}{sC} + R_{Th}} = \dfrac{1}{1 + \dfrac{R_{Th}}{sL \| \frac{1}{sC}}}$

Note: this is the response of the following circuit:

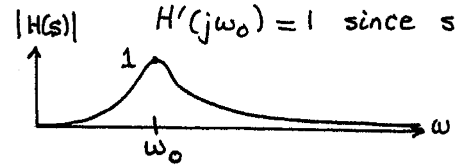


Note: We can sketch $H'(s)$ by considering

$\omega = 0$: $sL = 0 = $ wire, $\frac{1}{sC} = \infty = $ open
$H'(j0) = 0$ since V_o shorted by L

$\omega \to \infty$: $sL = \infty$ open, $\frac{1}{sC} = 0 = $ wire
$H'(j\infty) = 0$ since V_o shorted by C

$\omega = \omega_o$: $sL = -1/sC \Rightarrow sL \| 1/sC = {}^{L/C}/_{0} = \infty$
$H'(j\omega_o) = 1$ since $sL \| 1/sC = $ open



Back to $H'(s)$.

$$sL \parallel \frac{1}{sC} = \frac{L/C}{sL + \frac{1}{sC}} = \frac{L/C}{j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$H'(s) = \frac{1}{1 + \frac{R_{Th}}{\frac{L/C}{j(\omega L - \frac{1}{\omega C})}}} = \frac{1}{1 + j\frac{R_{Th}(\omega L - \frac{1}{\omega C})}{L/C}}$$

To find $\omega_{c1,2}$ we solve $|H'(s)| = \frac{1}{\sqrt{2}}$,

or $$\frac{1}{\left|1 + j\frac{R_{Th}(\omega L - \frac{1}{\omega C})}{L/C}\right|} = \frac{1}{\sqrt{2}}$$

or $$\sqrt{1^2 + \left[\frac{R_{Th}(\omega L - \frac{1}{\omega C})}{L/C}\right]^2} = \sqrt{2}$$

or $$\frac{R_{Th}(\omega L - \frac{1}{\omega C})}{L/C} = \pm 1$$

or $$R_{Th}\left(\omega L - \frac{1}{\omega C}\right) = \pm L/C$$

or $$\omega L - \frac{1}{\omega C} = \pm L/R_{Th}C$$

or $$\omega^2 - \frac{1}{LC} = \pm \frac{1}{R_{Th}C}\omega$$

or $$\omega^2 \pm \frac{1}{R_{Th}C}\omega - \frac{1}{LC} = 0$$

or $\omega_{c1,2} = \pm \frac{1}{2R_{Th}C} \pm \sqrt{\left(\frac{1}{2R_{Th}C}\right)^2 + \frac{1}{LC}}$

Since $\omega_{c1,2} > 0$ we use $+\sqrt{\ }$ only:

$$\omega_{c1,2} = \pm \frac{1}{2R_{Th}C} + \sqrt{\left(\frac{1}{2R_{Th}C}\right)^2 + \frac{1}{LC}}$$

Now for the numbers:

$$\frac{1}{2R_{Th}C} = \frac{1}{2 \cdot 5\|10 \cdot 400p} \text{ r/s}$$

$$= \frac{1}{2 \cdot 5 \cdot 1\|2 \cdot 400p} \text{ r/s}$$

$$= \frac{1}{2(5)\frac{2}{3} 400p} \text{ r/s}$$

$$= \frac{3}{2} \frac{1}{4k\,p} \text{ r/s}$$

$$\frac{1}{2R_{Th}C} = \frac{3}{8} \text{ G r/s}$$

$$\omega_0 = \frac{1}{2} \text{ G r/s} = \frac{4}{8} \text{ G r/s} \quad \text{from earlier}$$

$$\omega_{c1,2} = \pm \frac{3}{8} + \sqrt{\left(\frac{3}{8}\right)^2 + \left(\frac{4}{8}\right)^2} \text{ G r/s}$$

$$= \pm \frac{3}{8} + \frac{5}{8} \text{ G r/s}$$

$$\omega_{c1,2} = \frac{1}{4} \text{G and } 1 \text{G r/s}$$

c) $\beta \equiv$ bandwidth $= \omega_{c2} - \omega_{c1} = 2 \cdot \frac{1}{2 R_{Th} C}$

$$\beta = 2 \cdot \frac{3}{8} \text{ G r/s} = \frac{3}{4} \text{ G r/s}$$

d) $Q \equiv$ quality factor $= \frac{\omega_0}{\beta} = \frac{\frac{1}{2} \text{ G r/s}}{\frac{3}{4} \text{ G r/s}} = \frac{2}{3}$