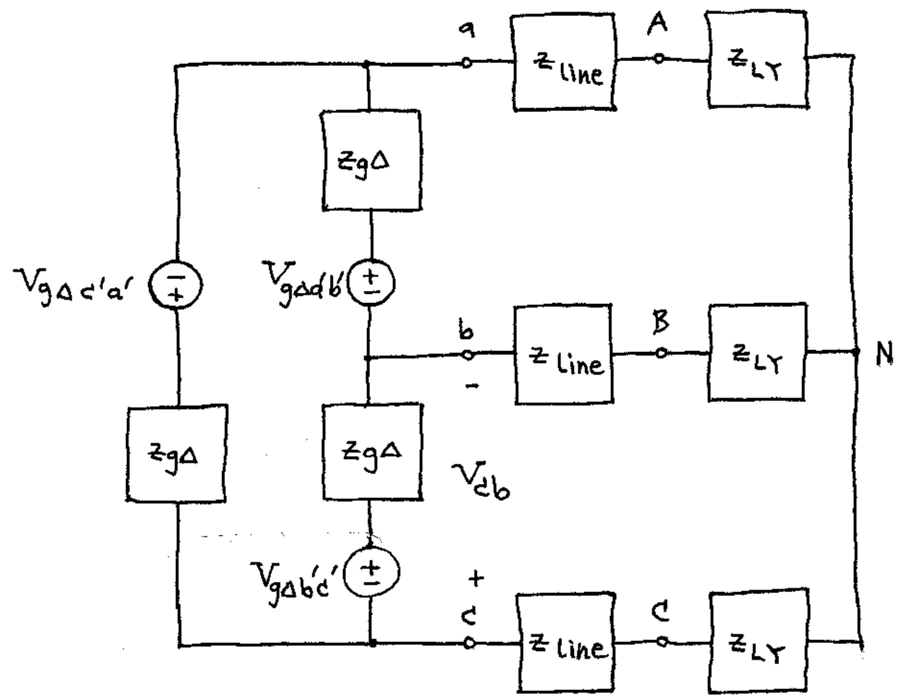
2.



$V_{g\Delta a'b'} = 116\angle 0°\ V$ $\qquad Z_{g\Delta} = 4.5 + j6\ \Omega$

$V_{g\Delta b'c'} = 116\angle -120°\ V$ $\qquad Z_{line} = 1.2 + j7.7\ \Omega$

$V_{g\Delta c'a'} = 116\angle +120°\ V$ $\qquad Z_{LY} = 4 - j3\ \Omega$

a) Draw a single-phase equivalent circuit

b) Find V_{cb}

sol'n: a) We convert the circuit to a Y-Y config.

For the generator, we have a Δ that we convert to a Y configuration.

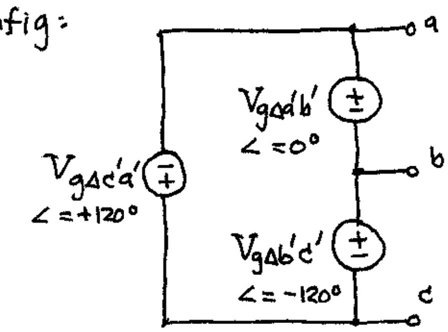
For Δ-to-Y transformation, we have

$$Z_{gY} = \frac{Z_{g\Delta}}{3} = \frac{4.5 + j6\ \Omega}{3}$$

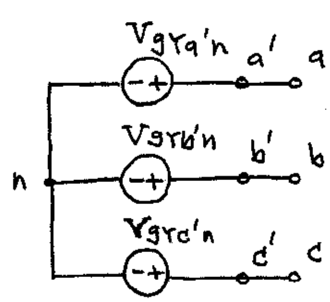
or $Z_{gY} = 1.5 + j2\ \Omega$

For the voltage sources, we assume $Z_{gY} = 0 = Z_{g\Delta}$ and find equivalent voltages for Δ and Y.

Δ config:



Y config:



For V_{ab} to be the same for both diagrams, we have

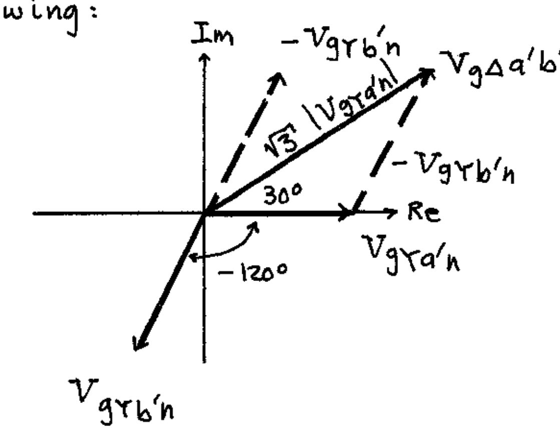
$$V_{g\Delta a'b'} = V_{gYa'n} - V_{gYb'n}$$

where $V_{gYb'n} = V_{gYa'n} \cdot 1\angle -120^\circ$

Thus,

$$V_{g\Delta a'b'} = V_{gYa'n}\,(1 - 1\angle -120^\circ)$$

Using a phasor diagram, we have the following:



Note: We treat $V_{gYa'n}$ as though it has $\angle = 0°$. This works because we are only looking for the relative value of $V_{g\Delta a'b'}$ given $V_{gYa'n}$. In other words, the eq'n we derive from the diagram is valid.

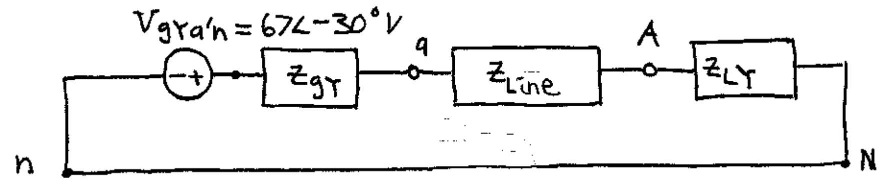
$$V_{g\Delta a'b'} = V_{gYa'n} \cdot \sqrt{3}\,\angle 30^\circ$$

or $$V_{gYa'n} = V_{g\Delta a'b'} \cdot \frac{1}{\sqrt{3}}\angle -30^\circ$$

or $$V_{gYa'n} = 116\angle 0^\circ \cdot \frac{1}{\sqrt{3}}\angle -30^\circ \text{ V}$$

$$V_{gYa'n} \doteq 67\angle -30^\circ \text{ V}$$

Single-phase equivalent:



$$Z_{gY} = 1.5 + j2\,\Omega$$
$$Z_{Line} = 1.2 + j7.7\,\Omega$$
$$Z_{LY} = 4 - j3\,\Omega$$

b) To find V_{cb}, we determine V_{an} and use a phasor diagram to relate V_{an} to V_{ab}. (But this is the same as the phasor diagram used in part (a).)

$$V_{ab} = V_{an} \cdot \sqrt{3} \angle 30°$$

Given V_{ab}, we have $V_{bc} = V_{ab} \cdot 1\angle -120°$

and $V_{cb} = -V_{bc} = V_{bc} \cdot 1\angle \pm 180°$.

Combining eq'ns, we have the following result:

$$V_{cb} = V_{an} \cdot \sqrt{3} \angle 30° \cdot 1\angle -120° \cdot 1\angle 180°$$

$$V_{cb} = V_{an}\ \sqrt{3} \angle 90°$$

What remains is to find V_{an} from the single-phase equivalent. We have a V-divider:

$$V_{an} = V_{aN} = V_{gYa'n} \cdot \frac{Z_{line} + Z_{LY}}{Z_{gY} + Z_{line} + Z_{LY}}$$

or $$V_{an} = 67\angle{-30^\circ}\,V \cdot \frac{1.2+j7.7+4-j3\ \Omega}{1.5+j2\,\Omega + 1.2+j7.7+4-j3\,\Omega}$$

$$= 67\angle{-30^\circ}\,V \quad \frac{5.2+j4.7}{6.7+j6.7}$$

$$= 67\angle{-30^\circ}\,V \quad \frac{5.2+j4.7}{6.7(1+j)}$$

$$= 67\angle{-30^\circ}\,V \cdot \frac{7\angle 42^\circ}{6.7(\sqrt{2})\angle 45^\circ}$$

$$= \frac{70}{\sqrt{2}}\angle{-33^\circ}\ V$$

Substituting into $V_{cb} = V_{an}\sqrt{3}\angle 90^\circ$ yields

$$V_{cb} = \frac{70}{\sqrt{2}}\angle{-33^\circ}\,V \cdot \sqrt{3}\angle 90^\circ$$

$$V_{cb} = 85.7\angle 57^\circ V \quad \text{or} \quad 70\sqrt{\frac{3}{2}}\angle 57^\circ V$$

or $V_{cb} = 51.4 + j68.6\ V$