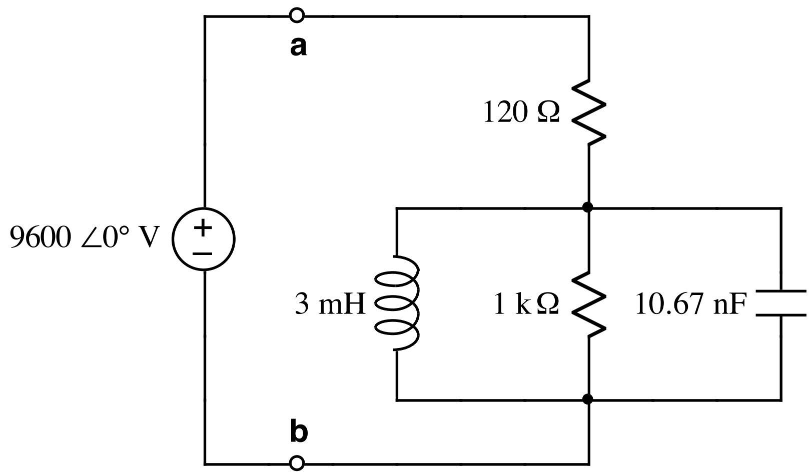
HOMEWORK #7 prob 1 solution

Ex:



Note: $\omega = 250$ kr/s.

Do the following for the impedance to the right of the a, b terminals:

a) Calculate complex power $S = P + jQ$.

b) Calculate average (or DC) power.

c) Calculate maximum instantaneous power.

d) Sketch the power waveform, $p(t)$.

SOL'N: a) The phasor, S, describes the sinusoidal part of the AC power. By coincidence, P happens to be both the real part of S and the value of the average or DC part of the AC power waveform. Thus, P is referred to as the average, DC, or real power. Q is the imaginary part of S and is referred to as the reactive power.

A convenient formula for S is based on the magnitude of current:

$$S = \frac{1}{2}|\mathbf{I}|^2 z$$

where $\mathbf{I}$ is the phasor current in the impedance, z, to the right of the a and b terminals.

We calculate z in the usual way:

$$z = 120\ \Omega + 1\ \text{k}\Omega \parallel j\omega \cdot 3\ \text{mH} \parallel \frac{1}{j\omega \cdot 10.67\ \text{nF}}$$

or

$$z = 120\ \Omega + 1\ \text{k}\Omega \parallel j250\ \text{kr/s} \cdot 3\ \text{mH} \parallel \frac{1}{j250\ \text{kr/s} \cdot 10.67\ \text{nF}}$$

or

$$z = 120\ \Omega + 1\ \text{k}\Omega \parallel j750\ \Omega \parallel -j375\ \Omega$$

or

$$z = 120\ \Omega + 1\ \text{k}\Omega \parallel (j375\ \Omega \cdot 2 \parallel -1)$$

or

$$z = 120\ \Omega + 1\ \text{k}\Omega \parallel -j750\ \Omega$$

or

$$z = 120\ \Omega + 250 \cdot 4 \parallel -j3$$

or

$$z = 120\ \Omega + 250 \cdot \frac{-j12}{4 - j3}$$

or

$$z = 120\ \Omega + -j120(4 + j3)$$

or

$$z = 480(1 - j)\ \Omega$$

Now we use Ohm's Law to find the phasor current:

$$\mathbf{I} = \frac{9600\angle 0^\circ\ \text{V}}{Z} = \frac{9600\angle 0^\circ\ \text{V}}{480(1 - j)\ \Omega} = \frac{20\angle 0^\circ\ \text{A}}{\sqrt{2}\angle -45^\circ} = \frac{20}{\sqrt{2}}\angle 45^\circ\ \text{A}$$

or

$$\mathbf{I} = 14.14\angle 45^\circ\ \text{A}$$

NOTE: We actually only need the magnitude of the current, which is somewhat easier to compute. Since we will also be squaring the magnitude, we may omit the calculation involving the square-root of two, as well.

We now calculate S:

$$S = \tfrac{1}{2}|\mathbf{I}|^2 z = \tfrac{1}{2}\left(\frac{20}{\sqrt{2}}\right)^2 480(1 - j) \text{ VA}$$

or

$$S = 48\text{k}(1 - j) \text{ VA} = 48\text{k} - j48\text{k VA} = 48\text{k}\sqrt{2}\angle -45° \text{ VA}$$

NOTE: We write the units of S as VA (volt·amps). This is mathematically the same as Watts, but it indicates that the quantity is S when it appears without being explicitly designated as S.

b) The average (or DC) power is the same as the real part of S, as noted above:

$$P = 48 \text{ kW}$$

c) The maximum instantaneous value of $p(t)$ occurs when the sinusoidal portion of the AC power reaches its peak value. At that point, we have the DC power plus the magnitude of the sinusoid:

$$\max_t p(t) = P + |S| = 48\text{k} + 48\text{k}\sqrt{2} \approx 116 \text{ kW}$$

d) The waveform is the DC power, P, plus the sinusoid for phasor S (at frequency 2ω): $p(t) = P + |S|\cos(2\omega t + \angle S)$.