HOMEWORK #5 prob 3.a solution

EX: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{2s - 20}{(s^2 + 4)}$$

SOL'N: A simple approach for finding the inverse transform is to recognize that this form corresponds to the sum of transformed cosine and sine functions with a denominator of $s^2 + 4 = s^2 + \omega^2$. That is, $\omega = 2$.

$$F(s) = \frac{2s - 20}{s^2 + 4} = A\frac{s}{s^2 + \omega^2} + B\frac{\omega}{s^2 + \omega^2}$$

If we equate the numerators on the left and right, we have the following result:

$$2s - 20 = As + B\omega = As + B \cdot 2$$

The coefficients of each power of s must match (including s^0, the constant term). Starting with the highest power of s, we match the coefficients on the left and right to determine the value of A and B.

$$A = 2 \text{ and } B = -10$$

The inverse transform follows directly from the above results:

$$\mathcal{L}^{-1}\left[\frac{2s - 20}{s^2 + 4}\right] = \mathcal{L}^{-1}\left[A\frac{s}{s^2 + 4} + B\frac{2}{s^2 + 4}\right] = A\cos 2t + B\sin 2t$$

or

$$\mathcal{L}^{-1}\left[\frac{2s - 20}{s^2 + 4}\right] = 2\cos 2t - 10\sin 2t$$