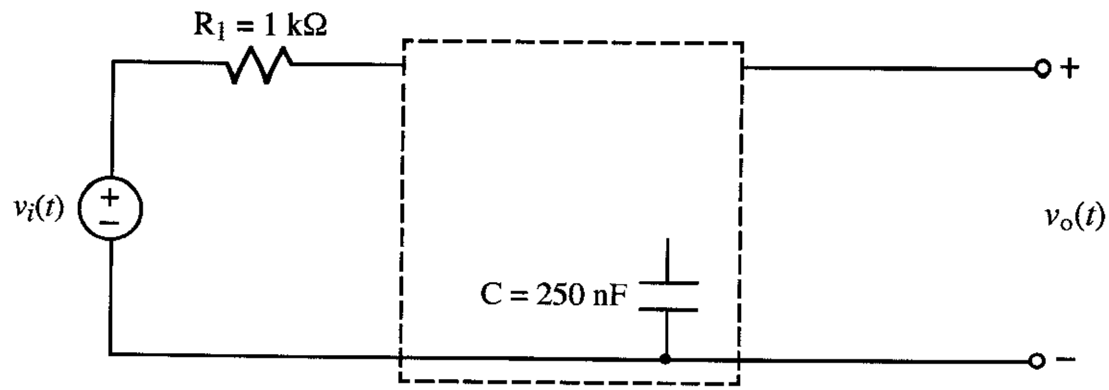
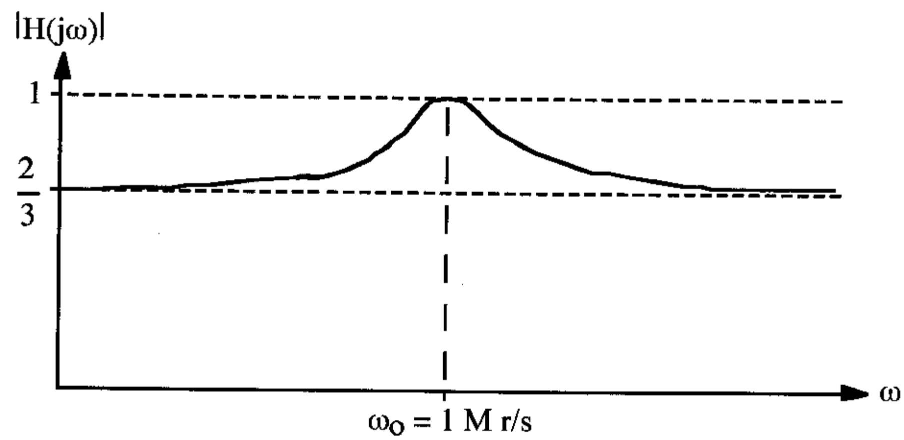
1. (30 points)





Given the capacitor connected as shown in the box and using not more than one each R and L, design a circuit to go in the dashed-line box that will produce the |H(jω)| vs. ω shown above, that is:

$$|H(j\omega)| = \frac{2}{3} \text{ at } \omega = 0$$

$$|H(j\omega)| = 1 \text{ at } \omega_o = 1 \text{ M rad/s}$$

$$|H(j\omega)| = \frac{2}{3} \text{ as } \omega \to \infty$$

sol'n: We have a modified band-pass filter. We want higher gain at resonance, implying that we want the components placed between the top and bottom rails to act like an open circuit at frequency ω_0.

An L and C in parallel act like an open circuit at frequency ω_0 where $\omega_0^2 = 1/LC$:

$$sL = -\frac{1}{sC} \quad \text{at } \omega_0 \Rightarrow sL \| \frac{1}{sC} = \frac{jX(-jX)}{jX - jX} = \frac{X^2}{0} \to \infty$$

Thus, we must have an L in parallel with the C.

$$\omega_0 = 1M \text{ r/s}, \quad \omega_0^2 = \frac{1}{LC} \quad \text{so} \quad L = \frac{1}{\omega_0^2 C}$$

$$\text{or} \quad L = \frac{1}{(1M)^2 \cdot 250n} H = \frac{1}{250k} H = \frac{4}{1M} H = 4\,\mu H$$

$$\boxed{L = 4\,\mu H}$$

When the L and C become an open circuit at ω_0, we want to have a gain of 1. Thus, any R that we add to the circuit must be in series with the LC or in series with R_1, as placing an R in parallel with L and C would eliminate the open circuit.

Now we consider $\omega = 0$ and $\omega = \infty$.

At $\omega = 0$, L = wire and C = open. wire$\|$open = wire.

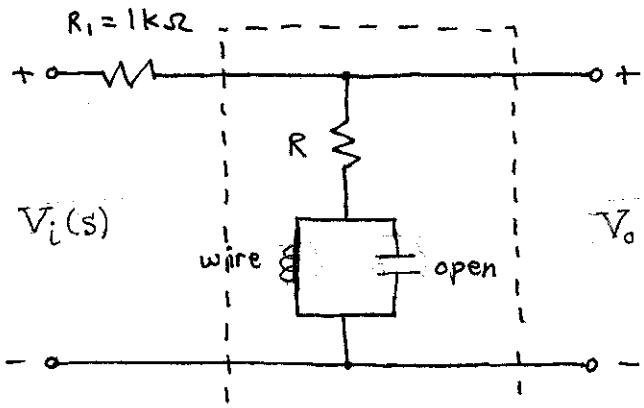
At $\omega \to \infty$, L = open and C = wire. open $\|$ wire = wire.

So the LC acts like a wire at both extremes.

If we only have a wire across the top and bottom rails, (i.e. across v_o), then $|H| = 0$.

Since $|H(j\omega)| = \frac{2}{3}$ at $\omega = 0$ and $\omega \to \infty$, we must place an R in series with the LC to create a voltage divider.

At $\omega = 0$ or $\omega \to \infty$ we have the following model:



$$|H(s)| = \left|\frac{V_o(s)}{V_i(s)}\right| = \left|\frac{R}{R+R_1}\right| = \frac{R}{R+R_1} = \frac{2}{3}$$

Solving for R yields $\boxed{R = 2k\Omega}$.

Contents of box: