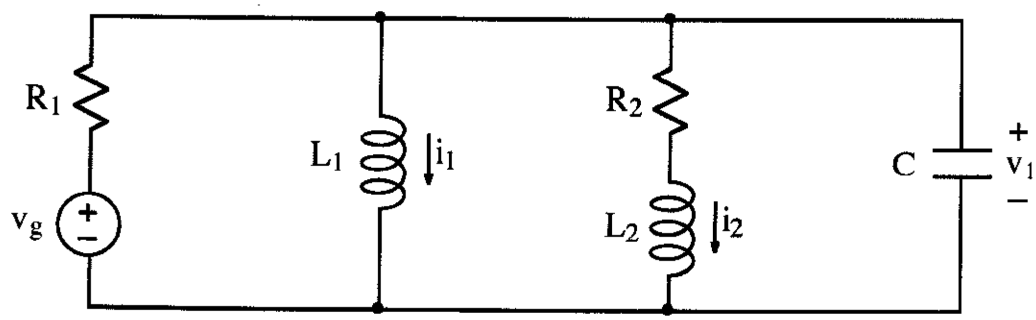
2. (30 points)



At t = 0, $v_g(t)$ switches instantly from $-v_o$ to v_o.

20 pts a) Write the state-variable equations for the circuit in terms of the state vector:

$$\vec{x} = \begin{bmatrix} i_1 \\ i_2 \\ v_1 \end{bmatrix}$$

10 pts b) Evaluate the state vector at $t = 0^+$.

soln: a) On the left side of each eq'n we must have $\frac{d}{dt}$ state var.

On the right side of each eq'n we must have only state vars and constants.

We start by using $\frac{di_L}{dt} = \frac{v_L}{L}$ and $\frac{dv_C}{dt} = \frac{i_C}{C}$.

$$\frac{di_1}{dt} = \frac{v_{L1}}{L_1} = \frac{v_1}{L_1} \quad \text{(since } L_1 \text{ parallel } C\text{)}$$

$$\frac{di_2}{dt} = \frac{v_{L2}}{L_2} = \frac{v_1 - i_2 R_2}{L_2} \quad \text{(v-loop on right)}$$

$$\frac{dv_1}{dt} = \frac{i_C}{C} = \frac{\frac{v_g - v_1}{R_1} - i_1 - i_2}{C}$$

Our official answer lacks the middle terms:

$$\frac{d}{dt} i_1 = \frac{v_1}{L_1}$$

$$\frac{d}{dt} i_2 = \frac{v_1 - i_2 R_2}{L_2}$$

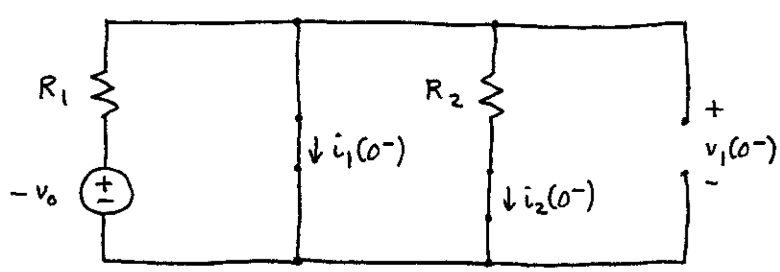
$$\frac{d}{dt} v_1 = \frac{\frac{v_g - v_1}{R_1} - (i_1 + i_2)}{C}$$

b) The state vector at $t = 0^+$

$=$ state vector at $t = 0^-$.

Reason? State variables i_L and v_C cannot change instantly.

At $t = 0^-$, $L =$ wire, $C =$ open, $v_g = -v_0$



All the current flows thru short formed by L_1.

$$i_1(0^-) = -v_0 / R_1$$

$i_2(0^-) = 0$ since all current flows thru L_1

$v_1(0^-) = 0$ since v_1 is across L_1 short

Thus, we have

$$\begin{bmatrix} i_1(0^-) \\ i_2(0^-) \\ v_1(0^-) \end{bmatrix} = \begin{bmatrix} -\frac{V_0}{R_1} \\ 0 \\ 0 \end{bmatrix}.$$