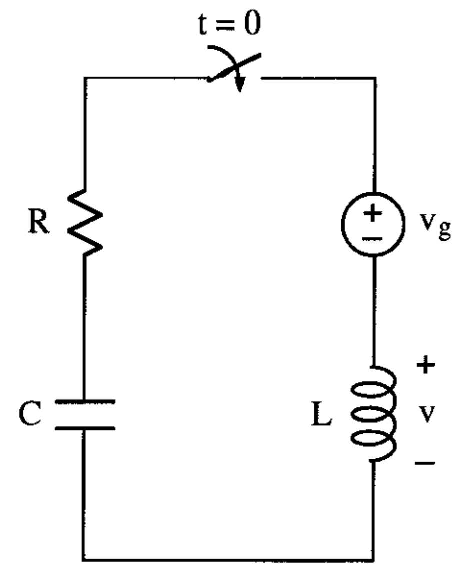
1. (30 points)



After being open for a long time, the switch closes at t = 0.

The inductor carries no current at time t = 0^-, and the capacitor stores no energy at $t = 0^-$.

20 pts a) Give expressions for the following in terms of v_g, R, L, and C:

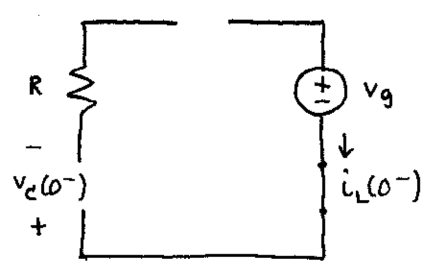
$v(t = 0^+)$ and $\left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$

10 pts b) Find the numerical values of L and the characteristic root, s, given the following information:

$C = 1$ nF $R = 20\ \Omega$ circuit is Critically Damped

sol'n: a) At $t = 0^-$, L acts like wire and C acts like open. We find $i_L(t=0^-)$ and $v_C(0^-)$, the energy vars.

$t = 0^-$:



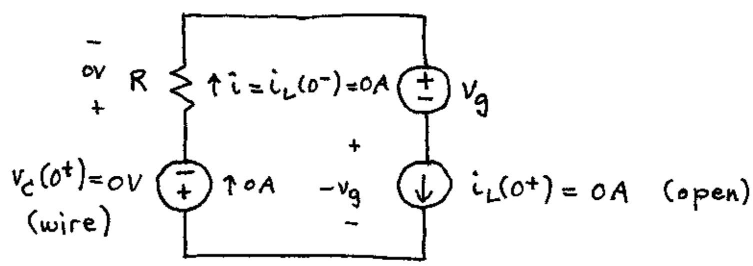
The problem states that $i_L(0^-) = 0$, which is what it must be with C = open.

From the circuit $v_C(0^-)$ could be anything, but

... the problem states that $v_c(0^-) = 0V$.

At $t=0^+$, we have $i_L(0^+) = i_L(0^-) = 0\,A$
and $v_c(0^+) = v_c(0^-) = 0V$.

We model $i_L(0^+)$ as a current source and $v_c(0^+)$ as a voltage source. Also, the switch is now closed.



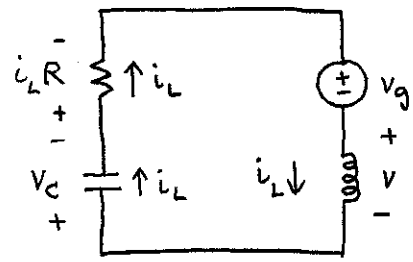
Since $i_L(0^+)$ flows around the loop, we can say all currents are 0. We can then find v-drops across all the components.

We find that $v_L(0^+) = -v_g = v(0^+)$

To find $\left.\dfrac{dv(t)}{dt}\right|_{t=0^+}$ we write an eq'n for v in terms of state (energy) vars i_L and v_c.

Note: We must write an eq'n that is true for all $t>0$. Thus, we use the original circuit diagram rather than the diagram for $t=0^+$. Use no numbers until after d/dt.

From the original circuit, we can write
$v = v_L = v_g + i_L R + v_c$:



Now we differentiate the entire eq'n for v:

$$\frac{dv}{dt} = \frac{d}{dt} v_g + R \frac{di_L}{dt} + \frac{dv_c}{dt}$$

$$\frac{dv}{dt} = 0 + R \frac{v_L}{L} + \frac{i_c}{C}$$

where we use $\frac{di_L}{dt} = \frac{v_L}{L}$, $\frac{dv_c}{dt} = \frac{i_c}{C}$.

$$\left.\frac{dv}{dt}\right|_{t=0^+} = R \frac{v_L(0^+)}{L} + \frac{i_c(0^+)}{C}$$

$$= R \frac{(-v_g)}{L} + \frac{0}{C}$$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = -v_g \frac{R}{L}$$

b) For a series RLC, we have

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = \frac{R}{2L}, \quad \omega_0^2 = \frac{1}{LC}$$

For critical damping, $\alpha^2 = \omega_0^2$.

$$\therefore \left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \qquad \text{or} \qquad \frac{R^2}{4L^2} = \frac{1}{LC}$$

$$\text{or} \quad \frac{R^2}{4L} = \frac{1}{C} \qquad \text{or} \qquad L = \frac{R^2 C}{4}$$

$$\text{or} \quad L = \frac{20^2 \Omega^2 \cdot 1\text{nF}}{4 \cdot} \qquad 100\ \text{nH}$$

Since $\sqrt{\alpha^2 - \omega_0^2} = 0$, we have $s = -\alpha = \frac{-R}{2L}$

$$\therefore s = -\frac{20\Omega}{2 \cdot 100\text{nH}} = -\frac{1}{10}\ \text{G}\ \text{r/s} = -100\text{M}\ \frac{\text{r}}{\text{s}}$$