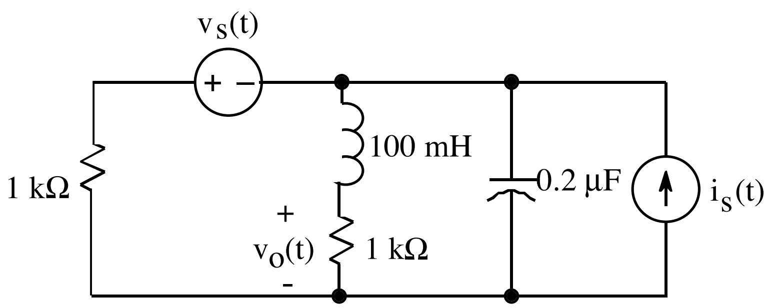
Ex:



$v_S(t) = 100 \cos(10^4 t)$ V

$i_S(t) = \sin(10^4 t)$ A

a) Find a numerical, time-domain expression for $v_O(t)$.

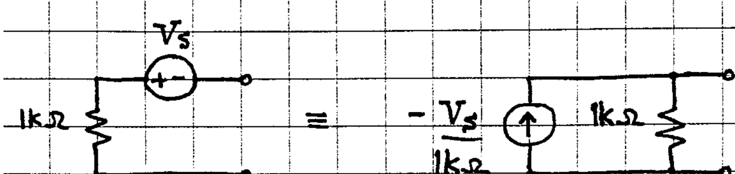
b) Show $\mathbf{V}_o$ on a phasor diagram.

Sol'n: a)

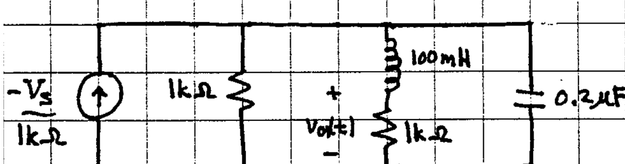
I'll use superposition. Other approaches such as node-V or mesh current would work, too.

circuit 1: $\mathbf{V}_s = 100\angle 0°$ V set $\mathbf{I}_s = 0$ A

Convert $\mathbf{V}_s$ and 1kΩ on left to Norton equivalent:

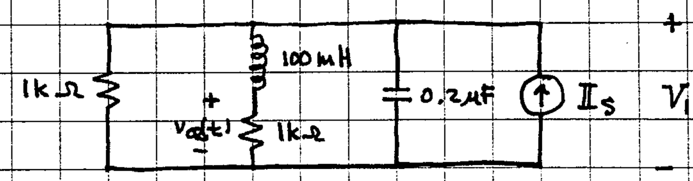


Our circuit 1 is now:



circuit 2: $\mathbf{V}_s = 0$ V and $\mathbf{I}_s = 1\angle -90°$ A (phasor for i_s)

Keeping the Norton equivalent, our circuit 2 is:



Clearly, we can sum the current sources from circuits 1 & 2 to find V_1 drop from top rail to bottom rail.

$$V_1 = \left(-\frac{V_s}{1k\Omega} + \mathbb{I}_s \right) 1k\Omega \,\|\, (j\omega L + 1k\Omega) \,\|\, \frac{-j}{\omega C}$$

$$j\omega L = j\, 10^4\, 100m = j\, 1k\Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{10^4\, 0.2\mu} = \frac{-j}{2\,m} = \frac{-j \cdot 1k\Omega}{2}$$

$$-\frac{V_s}{1k\Omega} + \mathbb{I}_s = \frac{-100 \angle 0^\circ\, V}{1k\Omega} + 1 \angle -90^\circ A$$

$$= -0.1 \angle 0^\circ A + 1 \angle -90^\circ A$$

$$= -0.1 - j$$

$$= \sqrt{0.1^2 + 1^2} \angle \tan^{-1} \frac{-1}{-0.1}$$

$$= 1.005 \angle -95.7^\circ$$

$$1k\Omega \,\|\, (j1k\Omega + 1k\Omega) \,\|\, -\frac{j}{2} 1k\Omega = 1k\Omega \frac{1}{\frac{1}{1} + \frac{1}{1+j} - \frac{2}{j}}$$

$$= \frac{1k}{1 + \frac{1-j}{2} + 2j} = \frac{1k}{\frac{3}{2} + j\frac{3}{2}} = \frac{2}{3}\,\frac{1k}{1+j}$$

$$= \frac{2}{3}k\,\frac{1-j}{2} = 1k\cdot\frac{(1-j)}{3}$$

$$V_1 = 1.005k\angle -95.7^\circ \cdot \frac{1-j}{3} \quad V$$

V_o from v-divider:

$$V_o = V_1 \cdot \frac{1k\Omega}{1k\Omega + j\omega L} = V_1 \cdot \frac{1k\Omega}{1k\Omega + j\,1k\Omega} = V_1\,\frac{1}{1+j} = V_1\,\frac{(1-j)}{2}$$

$$V_o = 1.005k\angle -95.7^\circ \left(\frac{1-j}{3}\;\frac{1-j}{2} = \frac{-j2}{6}\right) = \frac{1.005k\angle -185.7^\circ}{3}$$

$$V_o = 0.335k\angle -185.7^\circ \quad V$$

$$v_o(t) = 0.335k\cos\left(10^4 t - 185.7^\circ\right) = 0.335k\cos\left(10^4 t + 174.3^\circ\right) \; V$$

or $$v_o(t) = -0.335k\cos\left(10^4 t - 5.7^\circ\right) \; V$$

b)