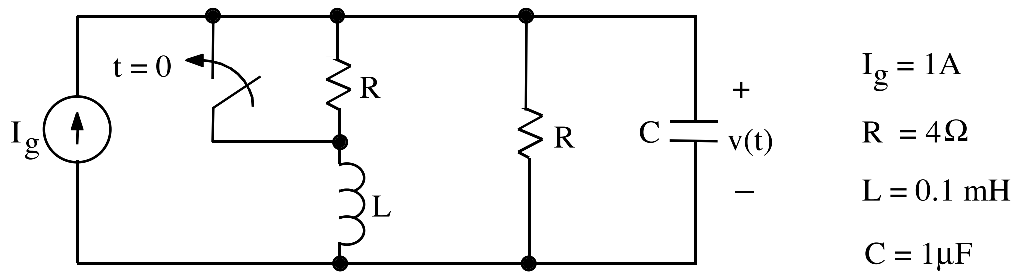
Ex:



The current source is a dc current source. After being open for a long time, the switch is closed at $t = 0$.

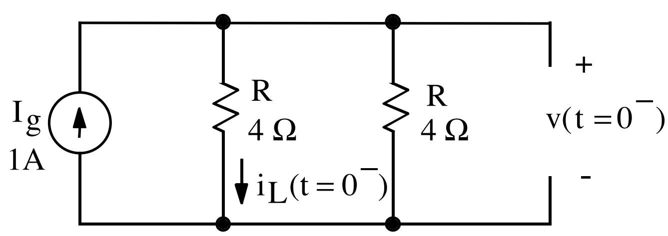
a) Write an expression for $V(s)$, the Laplace transform of $v(t)$.

b) From $V(s)$, the Laplace transform of $v(t)$, find the numerical values of $v(t)$ for $t = 0^+$ and $t \to \infty$.

c) By taking the inverse Laplace transform of $V(s)$, write a numerical time-domain expression for $v(t)$.

sol'n: (a) First we find initial conditions for L and C. (We need these for s-domain models of L and C.)

For $t = 0^-$, L acts like short, C acts like open circuit.

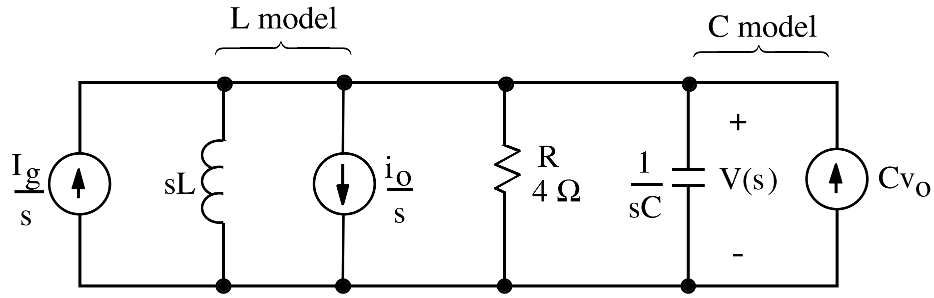


$$v(t = 0^-) = I_g \cdot R \| R = 1\text{A} \cdot 2\Omega = 2\text{V} \equiv v_o$$

$$i_L(t = 0^-) = I_g \cdot \frac{R}{R + R} = \frac{1}{2}\text{A} \equiv i_o$$

When we close the switch, we short out the first R.

s-domain model:



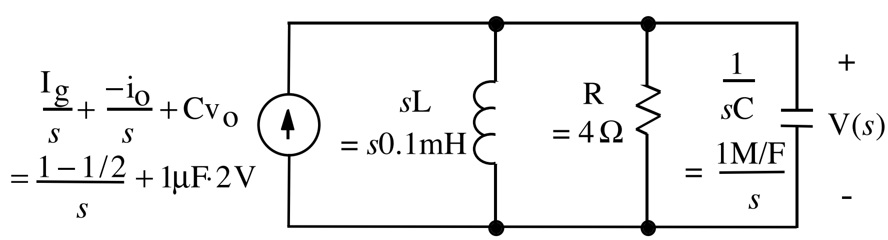
Note: A DC source corresponds to a step function <u>even if there is no switch and the source has the same output for all time.</u> Thus, we have I_g/s as the source in the *s*-domain. (Conceptually, we only need the current source for t > 0 because the initial conditions on L and C account for what the current source did for t < 0.)

$$\mathcal{L}\{I_g\} = \mathcal{L}\{I_g u(t)\} = \frac{1}{s}$$

Note: We may choose either a series *s*L and V-source for L or a parallel sL and I-source for L. Here, the parallel I-source model is more convenient. The same applies to the C.

Normally, we might use superposition at this point, turning on the I-sources one at a time and then summing currents or voltages to get a final answer.

Here, however, we have parallel I-sources that sum:



Combining the parallel impedances and using **V** = **I**z, we have

$$V(s) = \left(\frac{1}{2s} + 2\mu\text{FV}\right) \cdot sL \,||\, R \,||\, \frac{1}{sC}$$

To compute the parallel *z* value, we factor out numerators and use the following identity:

$$\frac{1}{a}\left\|\frac{1}{b}\right\|\frac{1}{c} = \frac{1}{a+b+c}$$

Thus, we factor out *sL* and *R*:

$$sL\|R\|\frac{1}{sC} = sLR \cdot \frac{1}{R}\left\|\frac{1}{sL}\right\|\frac{1}{sLRsC} = \frac{sLR}{R + sL + s^2 RLC}$$

Now divide by *RLC* to get denominator in proper form:

$$sL\|R\|\frac{1}{sC} = \frac{1}{C}\frac{s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

Check: Using the numerator and the first term in the denominator, we have the following units analysis:

$$\frac{s/C}{s^2} = \frac{1}{sC}$$

Thus, we have an impedance as we should have. The other terms in the denominator have the same units as s^2 since the units of s are, ironically, 1/sec or 1/s.

Now we plug in numbers to compute V(*s*):

$$\mathrm{V}(s) = \left(\frac{1}{2s} + 2\mu\right)\left(\frac{1\mathrm{M}\cancel{s}}{s^2 + \frac{1\mathrm{M}}{4}s + 10\mathrm{G}}\right)$$

where $\frac{1}{2s} + 2\mu = \frac{1 + s4\mu}{2\cancel{s}}$; $1\mathrm{M} = 1/\mathrm{C}$; $\frac{1\mathrm{M}}{4} = \frac{1}{\mathrm{RC}}$; $10\mathrm{G} = \frac{1}{\mathrm{LC}}$ (quadratic poles term)

Find poles for quadratic term in preparation for partial fractions:

$$s_{1,2} = -\frac{1\mathrm{M}}{8} \pm \sqrt{\left(\frac{1\mathrm{M}}{8}\right)^2 - 10\mathrm{G}}$$

$$\sqrt{\left(\frac{1\mathrm{M}}{8}\right)^2 - 10\mathrm{G}} = \sqrt{(125\mathrm{k})^2 - (100\mathrm{k})^2}$$

<u>not</u> complex poles

$s_{1,2}$ **=** -125k ± 75k rad/s (based on $5^2 - 4^2 = 3^2$ pythagorean triple)

$s_1 = -50\text{k}$, $s_2 = -200$ rad/s

Now use partial fractions:

$$V(s) = \frac{k_1}{s + 50\text{k}} + \frac{k_2}{s + 200\text{k}}$$

$$\text{k}_1 = \text{V}(s)(s + 50\text{k})\Big|_{s=-50\text{k}} = \frac{1 - 50\text{k} \cdot 4\mu}{2} \cdot \frac{1\text{M}}{-50\text{k} + 200\text{k}}$$

$$= \frac{1 - 200\text{m}}{2} \frac{1\text{M}}{150\text{k}}$$

$$= \frac{800\cancel{\text{m}}}{2} \frac{1\cancel{\text{M}}}{150\cancel{\text{k}}} = \frac{8}{3}$$

$$\text{k}_2 = \text{V}(s)(s + 50\text{k})\Big|_{s=-200\text{k}} = \frac{1 - 200\text{k}4\mu}{2} \frac{1\text{M}}{-200\text{k} + 50\text{k}}$$

$$= \frac{\cancel{\text{m}} \cdot 1\cancel{\text{M}}}{-2(150\cancel{\text{k}})} = -\frac{2}{3}$$

$$V(s) = \frac{8/3}{s + 50\text{k}} - \frac{2/3}{s + 200\text{k}}$$

sol'n: (b) Use the initial value theorem to find v(t=0^+):

$$v(t = 0^+) = \lim_{s \to \infty} sV(s) = \lim_{s \to \infty} s \frac{1 + s4\mu}{2} \frac{1M}{s^2 + \frac{1M}{4} s + 10G}$$

The largest power of s dominates in the numerator and in the denominator.

$$v(t = 0^+) = \lim_{s \to \infty} sV(s) = \lim_{s \to \infty} \frac{s^2 4\mu 1\text{M}}{2s^2} = \frac{4}{2} = 2\text{V} \ \surd$$

Note: We expect v(0^+) = 2V since this is the initial capacitor voltage.

Use the final value theorem to find v(t→∞):

$$\text{v}(\text{t} \to \infty) = \lim_{s \to 0} s\text{V}(s) = \lim_{s \to 0} s \frac{1 + s4\mu}{2} \frac{1\text{M}}{s^2 + \frac{1\text{M}}{4} s + 10\text{G}}$$

$$= 0 \cdot \frac{1}{2} \cdot \frac{1\text{M}}{10\text{G}} = 0\text{V} \ \checkmark$$

Note: We expect v(t) to decay, since L becomes a short.

sol'n: (c) From part (a) we have

$$V(s) = \frac{8/3}{s + 50\text{k}} - \frac{2/3}{s + 200\text{k}}.$$

Use the standard inverse Laplace transform term:

$$\mathcal{L}^{-1}\left\{\frac{k}{s + a}\right\} = ke^{-at}$$

This gives the final answer:

$$v(t > 0) = \frac{8}{3}e^{-50kt} - \frac{2}{3}e^{-200kt} \ V$$