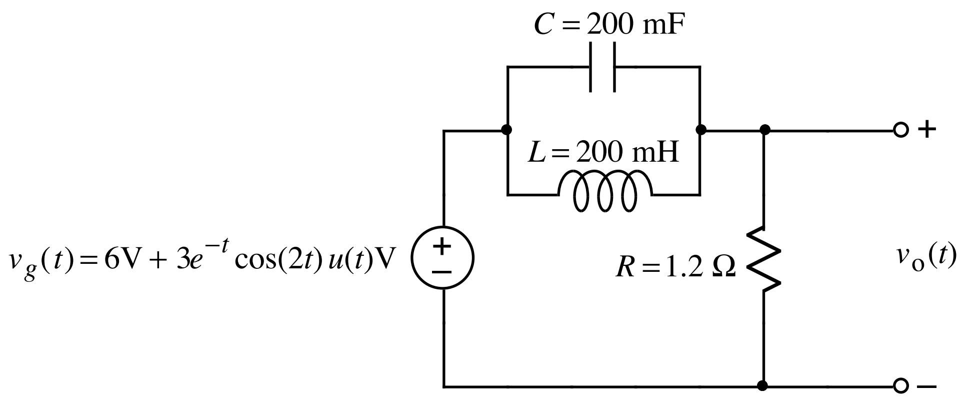
Ex:



Note: The 6 V in the $v_g(t)$ source is always on.

a) Write the Laplace transform, $V_g(s)$, of $v_g(t)$.

b) Draw the s-domain equivalent circuit, including source $V_g(s)$, components, initial conditions for L and/or C, and terminals for $V_o(s)$.

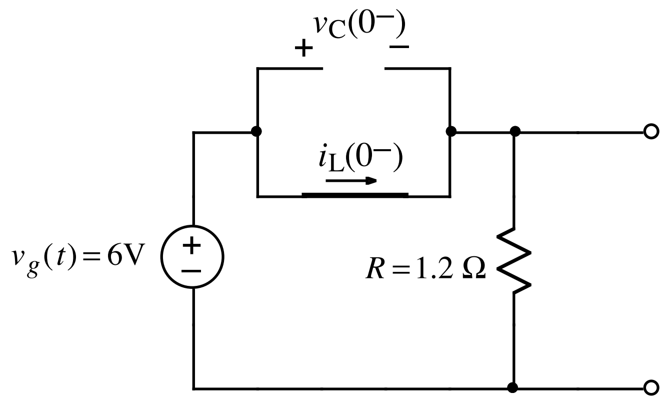
c) Write an expression for $V_o(s)$.

d) Apply the final value theorem to find $\lim_{t\to\infty} v_o(t)$.

SOL'N: a) We consider only the value of $v_g(t)$ for $t > 0$ when finding the Laplace transform:

$$\mathcal{L}\{v_g(t)\} = \mathcal{L}\left\{6u(t) + 3e^{-t}\cos(2t)u(t)\right\}\text{V} = \frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2}\text{V}$$

b) To find initial conditions, we assume that, since the circuit input is a constant 6 V, the circuit has reached a constant state where derivatives of voltages and currents are zero. Thus, we treat inductors as wires and capacitors as open circuits. We then find the energy variables, $i_L(t)$ and $v_C(t)$:

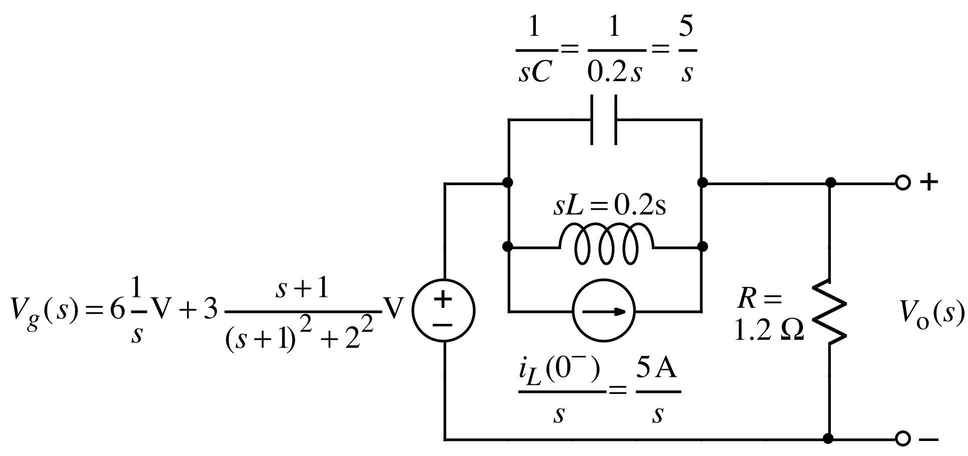


We have the following results:

$$i_L(0^-)=\frac{6\ \text{V}}{1.2\ \Omega}=5\ \text{A}$$

$$v_C(0^-)=v_L(0^-)=0\ \text{V}$$

We have a choice of whether to use a current source or a voltage source for the initial conditions on the L. (We may omit the initial condition source for the C, since the initial value is zero.) The choice made here is to use a parallel current course. Note that the current source corresponds to a step function in the time domain that produces current $i_L(0-)$ in the direction that $i_L(0-)$ should flow.



c) Using superposition, we turn on $V_g(s)$ and then $i_L(0^-)$ to find the total output signal $V_o(s)$:

$$V_o(s) = \left[\frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2}\right] \frac{R}{R + \frac{1}{sC}\Big\| sL} + \frac{i_L(0^-)}{s}\left[R \Big\| \frac{1}{sC} \Big\| sL\right]$$

or

$$V_o(s) = \left[\frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2}\right] \frac{1.2}{1.2 + \frac{1}{s0.2}\Big\| s0.2} + \frac{5}{s}\text{A}\left[1.2 \Big\| \frac{1}{s0.2} \Big\| s0.2\right]$$

d) The final value theorem statement is as follows:

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} sV_o(s)$$

or, in this case,

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} s\left(\left[\frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2}\right] \frac{R}{R + \frac{1}{sC}\Big\| sL} + \frac{i_L(0^-)}{s}\left[R \Big\| \frac{1}{sC} \Big\| sL\right]\right).$$

Unless we have zero divided by zero, we may evaluate each term as stated. We start by considering the value of the parallel L and C.

$$\lim_{s\to 0}\left(\frac{1}{sC}\Big\| sL\right) = \frac{1}{0}\Big\| 0 = 0 \text{ (anything in parallel with a short = short)}$$

Similarly, we have the parallel value of the R, L, and C:

$$\lim_{s\to 0}\left(R \Big\| \frac{1}{sC} \Big\| sL\right) = R \| 0 = 0$$

Substituting the above results into the final-value-theorem expression yields the following:

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} s\left(\left[\frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2}\right] \frac{R}{R + 0} + \frac{i_L(0^-)}{s}(0)\right)$$

or

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} s\left[\frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2}\right]$$

Now we multiply through by *s* and cancel factors of s^n common to numerator and denominator:

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0}\left[\frac{6s}{s} + \frac{s3(s+1)}{(s+1)^2 + 2^2}\right] = \lim_{s\to 0}\left[6 + \frac{s3(s+1)}{(s+1)^2 + 2^2}\right]$$

At this point, we may substitute $s = 0$ without creating a zero-divided-by-zero problem, and we obtain our result:

$$\lim_{t\to\infty} v_o(t) = 6 + \frac{0 \cdot 3(0+1)}{(0+1)^2 + 2^2} = 6 \text{ V}$$

This result makes sense, since only the 6V is left in $v_g(t)$ as time approaches infinity, and the inductor will act as a wire.