Ex:

\[ v_g(t) = 6V + 3e^{-t}\cos(2t)u(t)V \]

\[ C = 200 \text{ mF} \]
\[ L = 200 \text{ mH} \]
\[ R = 1.2 \text{ Ω} \]
\[ v_o(t) \]

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**Note:** The 6 V in the \( v_g(t) \) source is always on.

a) Write the Laplace transform, \( V_g(s) \), of \( v_g(t) \).

b) Draw the \( s \)-domain equivalent circuit, including source \( V_g(s) \), components, initial conditions for \( L \) and/or \( C \), and terminals for \( V_o(s) \).

c) Write an expression for \( V_o(s) \).

d) Apply the final value theorem to find \( \lim_{t \to \infty} v_o(t) \).

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**SOL'N:**

a) We consider only the value of \( v_g(t) \) for \( t > 0 \) when finding the Laplace transform:

\[
\mathcal{L}\{v_g(t)\} = \mathcal{L}\{6u(t) + 3e^{-t}\cos(2t)u(t)\} = \frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2} V
\]

b) To find initial conditions, we assume that, since the circuit input is a constant 6 V, the circuit has reached a constant state where derivatives of voltages and currents are zero. Thus, we treat inductors as wires and capacitors as open circuits. We then find the energy variables, \( i_L(t) \) and \( v_C(t) \):
We have the following results:

\[ i_L(0^-) = \frac{6 \text{ V}}{1.2 \text{ } \Omega} = 5 \text{ A} \]

\[ v_C(0^-) = v_L(0^-) = 0 \text{ V} \]

We have a choice of whether to use a current source or a voltage source for the initial conditions on the \( L \). (We may omit the initial condition source for the \( C \), since the initial value is zero.) The choice made here is to use a parallel current course. Note that the current source corresponds to a step function in the time domain that produces current \( i_L(0^-) \) in the direction that \( i_L(0^-) \) should flow.

\[ \frac{1}{sC} = \frac{1}{0.2s} = \frac{5}{s} \]

\[ V_g(s) = \frac{6}{s} \text{V} + \frac{s+1}{(s+1)^2 + 2^2} \text{V} \]

\[ i_L(0^-) = \frac{5 \text{ A}}{s} \]

\[ V_o(s) = \frac{1}{1.2 \text{ } \Omega} \]
c) Using superposition, we turn on \( V_g(s) \) and then \( i_L(0^-) \) to find the total output signal \( V_o(s) \):

\[
V_o(s) = \left[ \frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2} \right] \frac{R}{sC} \frac{1}{sL} + \frac{i_L(0^-)}{s} \left[ \frac{R}{sC} \frac{1}{sL} \right]
\]

or

\[
V_o(s) = \left[ \frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2} \right] \frac{1.2}{sC} \frac{1}{sL} + \frac{5}{s} \left[ \frac{1.2}{sC} \frac{1}{sL} \right].
\]

d) The final value theorem statement is as follows:

\[
\lim_{t \to \infty} v_o(t) = \lim_{s \to 0} sV_o(s)
\]

or, in this case,

\[
\lim_{t \to \infty} v_o(t) = \lim_{s \to 0} s \left( \frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2} \right) \frac{R}{sC} \frac{1}{sL} + \frac{i_L(0^-)}{s} \left[ \frac{R}{sC} \frac{1}{sL} \right].
\]

Unless we have zero divided by zero, we may evaluate each term as stated. We start by considering the value of the parallel \( L \) and \( C \).

\[
\lim_{s \to 0} \left( \frac{1}{sC} \frac{1}{sL} \right) = \frac{1}{0} = 0 \quad \text{(anything in parallel with a short = short)}
\]

Similarly, we have the parallel value of the \( R \), \( L \), and \( C \):

\[
\lim_{s \to 0} \left( \frac{1}{sC} \frac{1}{sL} \right) = \frac{R}{0} = 0
\]

Substituting the above results into the final-value-theorem expression yields the following:

\[
\lim_{t \to \infty} v_o(t) = \lim_{s \to 0} \left( \frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2} \right) \frac{R}{sR + 0} + \frac{i_L(0^-)}{s}(0)
\]

or
\[ \lim_{t \to \infty} v_\alpha(t) = \lim_{s \to 0} \left[ \frac{6}{s} + \frac{3(s+1)}{(s+1)^2 + 2^2} \right] \]

Now we multiply through by \( s \) and cancel factors of \( s^n \) common to numerator and denominator:

\[ \lim_{t \to \infty} v_\alpha(t) = \lim_{s \to 0} \left[ \frac{6s}{s} + \frac{s3(s+1)}{(s+1)^2 + 2^2} \right] = \lim_{s \to 0} \left[ 6 + \frac{s3(s+1)}{(s+1)^2 + 2^2} \right] \]

At this point, we may substitute \( s = 0 \) without creating a zero-divided-by-zero problem, and we obtain our result:

\[ \lim_{t \to \infty} v_\alpha(t) = 6 + \frac{0 \cdot 3(0+1)}{(0+1)^2 + 2^2} = 6 \text{ V} \]

This result makes sense, since only the 6V is left in \( v_\alpha(t) \) as time approaches infinity, and the inductor will act as a wire.