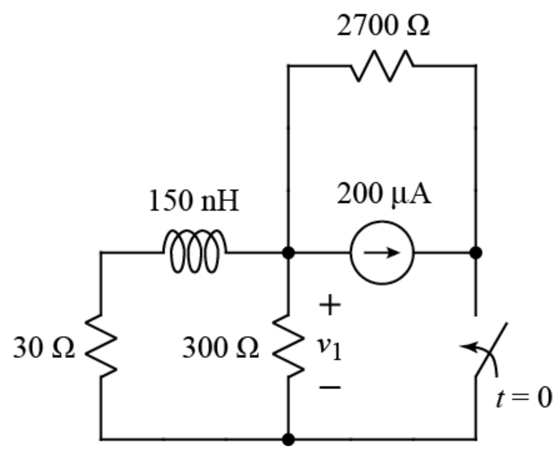
Ex:

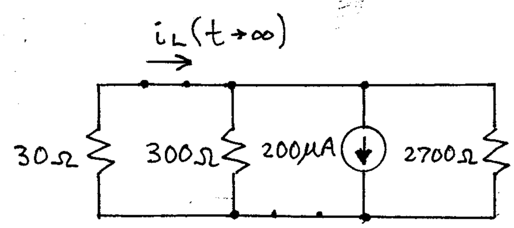


After being open for a long time, the switch closes at $t = 0$.

a) Calculate the energy stored on the inductor as $t \to \infty$.

b) Write a numerical expression for $v_1(t)$ for $t > 0$.

sol'n: a) For $t \to \infty$, we model the L as a wire and the switch is closed.



We a current-divider, with the 200μA from the current source splitting between the 30Ω and the 300Ω‖2700Ω.

Q. Is there a minus sign in the current-divider formula?

A. No. If we follow the arrow for i_L around to the 200μA source, it points in the same direction as the arrow in the 200 μA source.

$$i_L(t \to \infty) = 200\,\mu A \cdot \frac{300 \| 2700\,\Omega}{300 \| 2700\,\Omega + 30\,\Omega}$$

Now, $300 \| 2700\,\Omega = 300\,\Omega \cdot 1 \| 9 = 300 \cdot \frac{9}{10}$

$$= 270\,\Omega.$$

$$i_L(t \to \infty) = 200\,\mu A \cdot \frac{270\ \cancel{\Omega}}{270 + 30\ \cancel{\Omega}} = 200\,\mu A \cdot \frac{270}{300}$$

$$= \frac{540}{3}\,\mu A = 180\,\mu A$$

Energy stored on L is $\frac{1}{2} L i_L^2$:

$$w_L = \frac{1}{2} \cdot 150n\,(180\mu)^2\ J$$

$$= \frac{1}{2} \cdot 150n \cdot 32.4k\mu^2\ J$$

$$= \frac{1}{2} \cdot 4.86\,\mu k \mu \mu\ J \quad \text{since } 150n = 0.15\mu$$

$$\boxed{w_L = 2.43\ fJ} \qquad f \equiv \text{femto} = 10^{-15}$$

b) Using the circuit diagram from part (a) for $t \to \infty$, we see that $v_1(t \to \infty)$ is the voltage across all three resistors. The same voltage will be across the equivalent of the three resistors in parallel, and by Ohm's law the voltage will be the source current times the equivalent R.

$$v_1(t \to \infty) = -200\,\mu A \cdot 30\,\Omega \| 300\,\Omega \| 2700\,\Omega$$

$$= -200\,\mu A \cdot 30\,\Omega \| 270\,\Omega$$

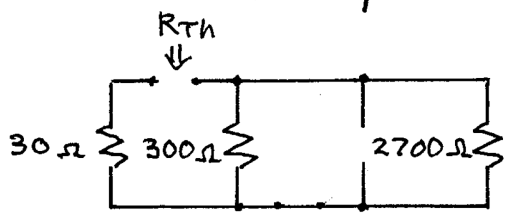
$$= -200\,\mu A \cdot 30\,\Omega \cdot 1 \| 9$$

$v_1(t\to\infty) = -200\,\mu A \cdot 27\,\Omega$

or

$v_1(t\to\infty) = -5.4\text{ mV}$

For R_{Th}, we use the circuit for $t>0$ with the L removed and the dependent source off.



$$R_{Th} = 30\,\Omega + 300\,\Omega \,\|\, 2700\,\Omega$$

$$= 30\,\Omega + 270\,\Omega$$

$$R_{Th} = 300\,\Omega$$

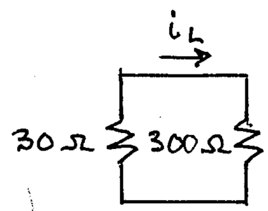
Our time constant is

$$\tau = \frac{L}{R_{Th}} = \frac{150\text{ nH}}{300\,\Omega} = 0.5\text{ ns}.$$

Last, we need $v_1(t=0^+)$. To find this value, we need a model for the L. To find a model for the L, we consider $t=0^-$, when the circuit is stable and the L acts like a wire.

The switch is open at $t=0^-$, which means the circuit loop on the lower left is connected to the circuit loop on the upper right by a single point.

No current can flow between the two loops in the circuit without causing an accumulation of charge. Thus, the two loops have no influence on each other. Thus, we need only consider the loop on the lower left.



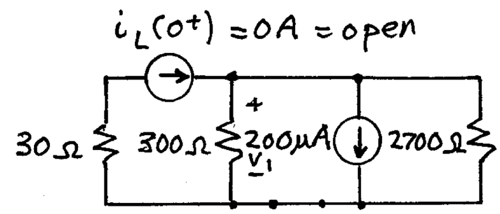
Since there is no power source,

$$i_L(0^-) = 0\ A.$$

Because i_L is an energy variable,

$$i_L(0^+) = i_L(0^-).$$

Now we can model the L as a current source at $t = 0^+$.



Because the L acts like an open, the 30Ω resistor dangles and has no impact on $v_1(t=0^+)$. $v_1(0^+)$ is given by the current source times the parallel resistance of 300Ω and 2700Ω, which is 270Ω.

$$v_1(0^+) = -200\,\mu A \cdot 270\,\Omega = -54\,mV$$

We use the general form of solution to finish the problem.

$$v_1(t>0) = v_1(t\to\infty) + [v_1(0^+) - v_1(t\to\infty)]e^{-t/\tau}$$

or

$$v_1(t>0) = -5.4\,mV + [-54\,mV - -5.4\,mV]\,e^{-t/0.5ns}$$

or

$$\boxed{v_1(t>0) = -5.4 - 48.6\,e^{-t/0.5ns}\ mV}$$