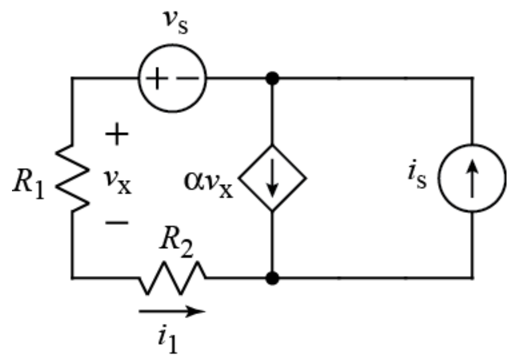
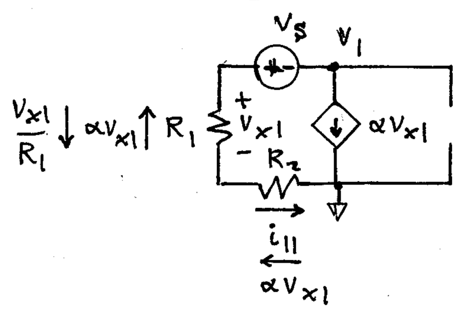
Ex:



Using superposition, derive an expression for i_1 that contains no circuit quantities other than i_S, v_S, R_1, R_2, and α. Note: $\alpha > 0$.

sol'n: We turn on one independent source at a time. Dependent sources are always on.

case I: v_S on, i_S off = open



Because of the open circuit, the current αv_{x1} flows thru R_2 and <u>up</u> thru R_1.

The current flowing <u>down</u> thru R_1 is v_{x1}/R_1.

$$\frac{v_{x1}}{R_1} = -\alpha v_{x1}$$

The only possible sol'n is $v_{x1} = 0$, $i_{11} = 0A$.

Or we can use the node-voltage method.

v_1 node: $\dfrac{v_1 + v_s}{R_1 + R_2} + \alpha v_{x1} = 0A$

where $v_{x1} = (v_1 + v_s)\dfrac{R_1}{R_1+R_2}$

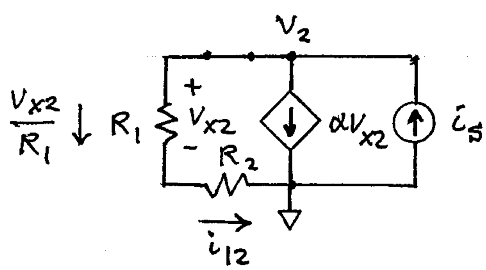
So $(v_1+v_s)\left(\dfrac{1+\alpha R_1}{R_1+R_2}\right) = 0A$

Since $\alpha > 0$, $\dfrac{1+\alpha R_1}{R_1+R_2} \neq 0$.

Thus, $v_1 + v_s = 0V$ or $v_1 = -v_s$.

This means the voltage drop across $R_1 + R_2$ is 0V, giving $i_{l1} = 0A$.

case II: v_s off = wire, i_s on



Using the node-voltage method, we have a current sum at v_2:

$$\frac{v_2}{R_1+R_2} + \alpha v_2 \frac{R_1}{R_1+R_2} - i_s = 0A$$

or $v_2 \dfrac{1+\alpha R_1}{R_1+R_2} = i_s$

or $v_2 = i_s \dfrac{R_1+R_2}{1+\alpha R_1}$

$$i_{12} = \frac{V_2}{R_1 + R_2} = \frac{i_s}{1 + \alpha R_1}$$

Or we could use a current sum directly in terms of v_x:

$$i_s = \frac{v_{x2}}{R_1} + \alpha v_{x2} = v_{x2}\left(\frac{1}{R_1} + \alpha\right)$$

or

$$v_{x2} = i_s \frac{R_1}{1 + \alpha R_1}$$

$$i_{12} = \frac{v_{x2}}{R_1} = \frac{i_s}{1 + \alpha R_1}$$

The total i_1 is the sum of i_{11} and i_{12}.

$$i_1 = i_{11} + i_{12} = 0 + i_{12} = \frac{i_s}{1 + \alpha R_1}$$

$$\boxed{i_1 = \frac{i_s}{1 + \alpha R_1}}$$