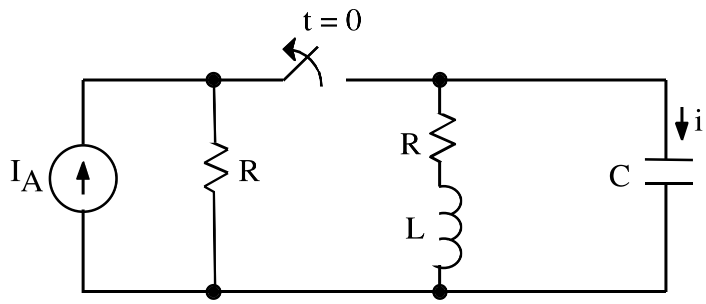
Ex:



$I_A = 1$ A
$R = 2400\ \Omega$
$L = 200\ \mu$H
$C = 50$ pF

a) After being closed for a long time, the switch is opened at t = 0. Give the values of the characteristic roots for the circuit and state whether *i*(t) is underdamped, overdamped, or critically damped.

b) Write a numerical time-domain expression for *i*(t), the current through the capacitance. This expression must not contain any complex numbers.

SOL'N: a) When the switch is open, we have series RLC.

$$\alpha = \frac{R}{2L} = \frac{2.4\text{k}}{2.200\mu} = \frac{1.4k}{400}\text{M} = 6\text{M}/s$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{200\mu\ 50\text{p}} = \frac{1}{10k\ \mu\text{p}} = \frac{1\text{M}\cdot\text{M}\cdot\text{M}}{10\text{k}} = 100\ \text{M}^2/s^2$$

$$\therefore \quad \omega_\text{o} = 10\ \text{M}/s$$

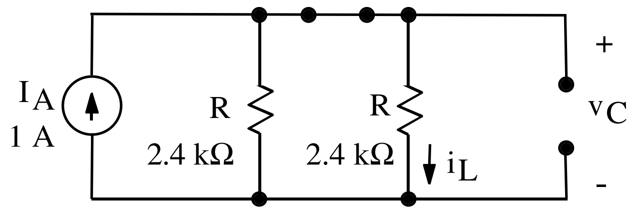
$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{(10\ \text{M})^2 - (6\ \text{M})^2} = 8\ \text{M}/s \qquad \left(6^2 + 8^2 = 10^2\right)$$

We have an oscillatory solution, and the circuit is underdamped.

b) Now find initial condition (i.e. i and di/dt, or v and dv/dt at t = 0^+).

$$\left.\begin{array}{l} i_L(t=0^+) = i_L(t=0^-) \\ v_C(t=0^+) = v_C(t=0^-) \end{array}\right\} \text{cannot change instantly}$$

Circuit for t = 0^-: L's = wires, C's = open circuits.



$i_L(t=0^-) = \frac{1}{2}$ A   (current divider)

$$v_C(t=0^-) = I_A R \,||\, R = I_A \cdot \frac{R}{2} = 1\text{A} \cdot 1.2\text{ k}\Omega = 1.2\text{ kV}$$

$$\therefore \quad i_L(t=0^+) = \frac{1}{2}\text{A}, \, v_C(t=0^+) = 1.2\text{ kV}$$

After the switch is open, $i = -i_L$ since C and L are in series.

$\therefore$ Solve for i_L and then change the sign. Note that i_L is the variable in the differential equation for a series *RLC*. Thus, we know how to find it.

We also need

$$\left.\frac{di_L(t)}{dt}\right|_{t=0^+}.$$

Use V-loop for RLC at $t = 0^+$:

$$L\frac{di_L}{dt} + i_L R - v_C = 0\text{ V}.$$

Note that at $t = 0^+$, ($i_R = i_L$ since R, L in series),

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{-i_L(t=0^+)R + v_C(t=0^+)}{L}$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{-\frac{1}{2}A \cdot 2.4\text{ k}\Omega + 1.2\text{ k}\Omega}{200\,\mu\text{H}}$$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = 0\text{ A}/s$$

Now use general underdamped solution:

$$i_L(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t)e^{-\alpha t}$$

$$B_1 = i_L(t=0^+), \qquad -\alpha B_1 + \omega_d B_2 = \left.\frac{di_L}{dt}\right|_{t=0^+} = 0 \text{ A}/s$$

$$\therefore \quad B_1 = \frac{1}{2}\text{ A}, \qquad B_2 = \frac{\alpha B_1}{\omega_d} = \frac{6M}{8M}\,\frac{1}{2}\text{ A} = \frac{3}{8}\text{ A}$$

$$\therefore \quad -i(t>0) = \left(\frac{1}{2}\cos 8\text{M}t + \frac{3}{8}\sin 8\text{M}t\right)e^{-6\text{M}t}\text{ A}$$