Ex:

\[ I_A = 1 \text{ A} \]
\[ R = 2400 \Omega \]
\[ L = 200 \mu \text{H} \]
\[ C = 50 \text{ pF} \]

a) After being closed for a long time, the switch is opened at \( t = 0 \). Give the values of the characteristic roots for the circuit and state whether \( i(t) \) is underdamped, overdamped, or critically damped.

b) Write a numerical time-domain expression for \( i(t) \), the current through the capacitance. This expression must not contain any complex numbers.

**Sol’N:**

a) When the switch is open, we have series RLC.

\[ \alpha = \frac{R}{2L} = \frac{2.4k}{2.200\mu} = \frac{1.4k}{400} \text{ M} = 6\text{M/s} \]

\[ \omega_0^2 = \frac{1}{LC} = \frac{1}{200\mu \cdot 50p} = \frac{1}{10k \mu \cdot M} = \frac{1\text{M} \cdot \text{M}}{10k} = 100 \text{ M}^2/\text{s}^2 \]

\[ \therefore \omega_0 = 10 \text{ M/s} \]

\[ \omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(10 \text{ M})^2 - (6 \text{ M})^2} = 8 \text{ M/s} \quad \left(6^2 + 8^2 = 10^2 \right) \]

We have an oscillatory solution, and the circuit is underdamped.

b) Now find initial condition (i.e. \( i \) and \( di/dt \), or \( v \) and \( dv/dt \) at \( t = 0^+ \)).

\[ I_L(t = 0^+) = I_L(t = 0^-) \]
\[ V_C(t = 0^+) = V_C(t = 0^-) \]

Cannot change instantly

Circuit for \( t = 0^- \): L's = wires, C's = open circuits.
\[ i_L(t=0^+) = \frac{1}{2} \text{A} \quad \text{(current divider)} \]

\[ v_C(t = 0^-) = I_A R \parallel R = I_A \cdot \frac{R}{2} = 1 \text{A} \cdot 1.2 \text{kΩ} = 1.2 \text{kV} \]

\[ : \quad i_L(t = 0^+) = \frac{1}{2} \text{A}, \; v_C(t = 0^+) = 1.2 \text{kV} \]

After the switch is open, \( i = -i_L \) since \( C \) and \( L \) are in series.

\[ : \quad \text{Solve for } i_L \text{ and then change the sign. Note that } i_L \text{ is the variable in the differential equation for a series } RLC. \text{ Thus, we know how to find it.} \]

We also need

\[ \left. \frac{di_L(t)}{dt} \right|_{t=0^+}. \]

Use V-loop for RLC at \( t = 0^+ \):

\[ L \frac{di_L}{dt} + i_L R = v_C = 0 \text{ V}. \]

Note that at \( t = 0^+ \), \( i_R = i_L \) since \( R, L \) in series).

\[ \left. \frac{di_L}{dt} \right|_{t=0^+} = \frac{-i_L(t=0^+)R + v_C(t=0^+)}{L} \]

\[ \left. \frac{di_L}{dt} \right|_{t=0^+} = \frac{-\frac{1}{2} \text{A} \cdot 2.4 \text{kΩ} + 1.2 \text{kΩ}}{200 \mu\text{H}} \]

\[ \left. \frac{di_L}{dt} \right|_{t=0^+} = 0 \text{ A/s} \]

Now use general underdamped solution:

\[ i_L(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t} \]
\[ B_1 = i_L(t = 0^+), \quad -\alpha B_1 + \omega_d B_2 = \left. \frac{di_L}{dt} \right|_{t=0^+} = 0 \text{ A/s} \]

\[ \therefore B_1 = \frac{1}{2} \text{ A, } \quad B_2 = \frac{\alpha B_1}{\omega_d} = \frac{6M}{8M} \frac{1}{2} \text{ A} = \frac{3}{8} \text{ A} \]

\[ \therefore -i(t > 0) = \left( \frac{1}{2} \cos 8Mt + \frac{3}{8} \sin 8Mt \right) e^{-6Mt} \text{ A} \]