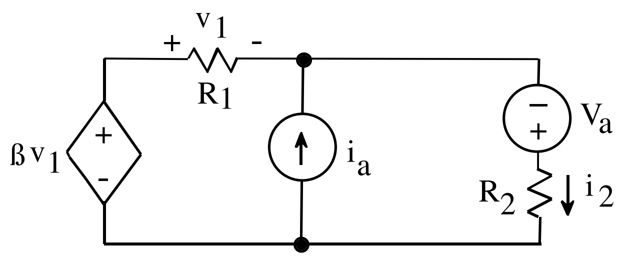
PRACTICE EXAM SOLUTION Prob 3

3. (30 points)

a. Derive an expression for i_2. The expression must not contain more than the circuit parameters ß, V_a, i_a, R_1, and R_2.

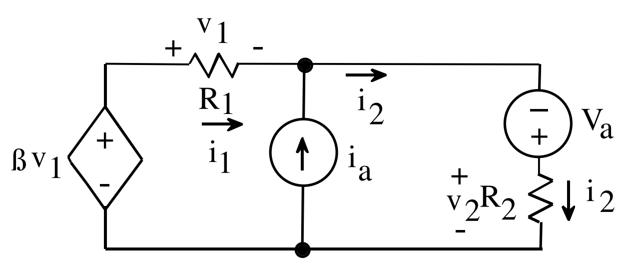


b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

ans: a) $i_2 = \dfrac{(1-\beta)i_aR_1 + V_a}{(1-\beta)R_1 + R_2}$

b) Many possible answers. See solution below.

sol'n: (a) Use Kirchhoff's laws to write several equations. Then eliminate unwanted variables.



We sum currents out of top-center node:

$$-i_1 - i_a + i_2 = 0$$

Note that summing currents out of bottom-center node does not give us anything new. By Ohm's law, we also have

$$i_1 = \frac{v_1}{R_1}$$

Now, we sum voltages around a loop. We choose the outer loop because the inner loops have a current source with unknown voltage drop.

$$\beta v_1 - v_1 + V_a - v_2 = 0 \quad \text{or} \quad (\beta - 1)v_1 + V_a - v_2 = 0$$

By Ohm's law, we also have

$$v_2 = i_2 R_2$$

After the Ohm's law substitutions, we have two equations, and we may eliminate v_1.

Use the simpler equation first:

$$\frac{v_1}{R_1} + i_a - i_2 = 0 \quad \text{or} \quad v_1 = (i_2 - i_a)$$

Substitute for v_1 in the second equation:

$$(\beta - 1)R_1(i_2 - i_a) + V_a - i_2 R_2 = 0$$

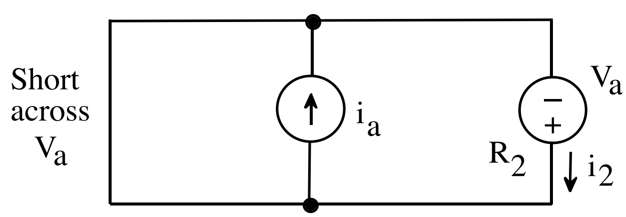
After some algebra, we get

$$i_2 = \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2} \quad \text{units consistent} \quad ✓$$

(b) There are many possible consistency checks.

1) $i_a = 0$ and $R_1 = 0$. Then $v_1 = 0$, $ßv_1 = 0$, and sum v's around outer loop gives $i_2 = V_a/R_2$. Our formula also gives V_a/R_2. ✓

2) Consider $R_1 = 0$ and $R_2 = 0$. As in (1), $v_1 = 0$ and $ßv_1 = 0$. Since $R_2 = 0$ we also end up with a short across V_a:



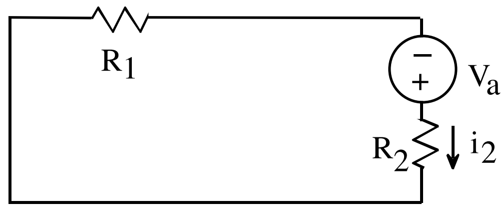
We expect $i_2 \to \infty$ for short across V_a

Our formula gives

$$\lim_{R_2 \to 0} \frac{V_a}{R_2} = \infty \qquad \text{for } i_2$$

from (1) ✓

3) Consider $i_a = 0$, $ß = 0$:

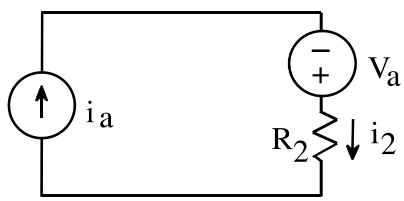


$$i_2 = \frac{V_a}{R_1 + R_2} \text{ by Ohm's law}$$

Our formula gives

$$i_2 = \frac{V_a}{R_1 + R_2} \quad ✓$$

4) Consider $R_1 \to \infty$ (open circuit)



Clearly $i_a = i_2$ since the same current flows through elements in series.

Our formula gives:

$$i_2 = \lim_{R_1 \to \infty} \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2} = \lim_{R_1 \to \infty} \frac{(1-\beta)i_a R_1}{(1-\beta)R_1} \quad ✓$$

$$= i_a$$

5) Consider $R_2 \to \infty$ (open circuit): We have $i_2 = 0$ since no current flows through the open circuit. Our formula gives:

$$i_2 = \lim_{R_1 \to \infty} \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2} = \frac{\text{const}}{\infty} = 0 \quad ✓$$

Many more consistency checks are possible.