Ex: Find the value of each of the following:

\[ R_1 = 2k \ \Omega \]
\[ R_2 = 3k \ \Omega \]
\[ R_3 = 2k \ \Omega \]
\[ R_4 = 3k \ \Omega \]
\[ R_5 \]

a) The above circuit operates in linear mode. Derive a symbolic expression for \( v_o \). The expression must contain not more than the parameters \( v_{s1}, v_{s2}, R_1, R_2, R_3, R_4, \) and \( R_5 \).

b) If \( v_{s1} = 0 \ \text{V} \) and \( v_{s2} = 1 \ \text{V} \), find the value of \( R_5 \) that will yield an output voltage of \( v_o = 1 \ \text{V} \).

c) Derive a symbolic expression for \( v_o \) in terms of common mode and differential input voltages:
\[
\frac{v_{cm}}{2} = \frac{v_{s2} + v_{s1}}{2} \quad \text{and} \quad \frac{v_{dm}}{2} = v_{s2} - v_{s1}
\]
The expression must contain not more than the parameters \( v_{cm}, v_{dm}, R_1, R_2, R_3, R_4, \) and \( R_5 \). Write the expression as \( v_{cm} \) times a term plus \( v_{dm} \) times a term. Hint: start by writing \( v_{s1} \) and \( v_{s2} \) in terms of \( v_{cm} \) and \( v_{dm} \):
\[
v_{s1} = v_{cm} - \frac{v_{dm}}{2} \quad \text{and} \quad v_{s2} = v_{cm} + \frac{v_{dm}}{2}
\]

d) Find the numerical value of the circuit's input resistance, \( R_{in} \), as seen by source \( v_{s2} \). In other words, write a formula for voltage, \( v_{s2} \), divided by \( i_2 \):
\[
R_{in} = \frac{v_{s2}}{i_2}
\]

Sol'n: a) Converting the circuitry comprised of \( v_{s1}, R_1, \) and \( R_2 \) into a Thevenin equivalent yields a standard differential amplifier.
Likewise, converting the circuitry comprising $v_s2$, $R_3$, and $R_4$ into a Thevenin equivalent yields further simplification:

$$R_{Th1} = \frac{2k\Omega \times 3k\Omega}{2k\Omega + 3k\Omega} = 1.2k\Omega$$

Since no current flows into the + input of the op-amp, $R_{Th2}$ has no voltage drop and may be removed.
Now, applying superposition reveals that this circuit is a combination of a standard negative-gain circuit and positive-gain circuit.

case I: \(v_{s1} \text{ on}, \ v_{s2} \text{ off} = \text{wire}\)

For this circuit, \(v_{1-} = v_{1+} = 0 \text{ V.} \) (The "1" stands for "case I"). Using \(v_{1-}\) to calculate \(i_{left1}\) and \(i_{right1}\) gives the following equation:

\[
i_{left1} = \frac{v_{s1} \cdot R_2}{R_1 + R_2} = \frac{-v_{o1}}{R_5} = i_{right1}
\]
\[ v_{o1} = -v_{s1} \frac{R_5}{R_1} \]

case II: \((v_{s1} \text{ off} = \text{wire}, v_{s2} \text{ on})\)

\[ R_{Th1} = \frac{R_1}{R_1 \parallel R_2} \]

For this circuit, \(v_{2-} = v_{2+} = v_{s2} \frac{R_4}{R_3 + R_4}\). (The "2" stands for "case II").

Using \(v_{2-}\) to calculate \(i_{left2}\) and \(i_{right2}\) gives the following equation:

\[ i_{left2} = \frac{v_{s2} \cdot \frac{R_4}{R_3 + R_4}}{\frac{R_1}{R_1 \parallel R_2}} = \frac{v_{s2} \cdot \frac{R_4}{R_3 + R_4} - v_{o2}}{\frac{R_5}{R_1 \parallel R_2}} = i_{right2} \]

or

\[ v_{o2} = v_{s2} \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_5}{\frac{R_1}{R_1 \parallel R_2}}\right) \]

Summing results gives the final result.

\[ v_o = -v_{s1} \frac{R_5}{R_1} + v_{s2} \frac{R_4}{R_3 + R_4} \left(1 + \frac{R_5}{\frac{R_1}{R_1 \parallel R_2}}\right) \]

Node voltage may also be used. For the \(v_-\) node:

\[ \frac{v_-}{R_4} + \frac{v_- - v_{s2}}{R_3} = 0 \text{ A} \]

For the \(v_+\) node:
\[
\frac{v_- - v_{s1}}{R_1} + \frac{v_-}{R_2} + \frac{v_- - v_o}{R_5} = 0 \text{ A}
\]

Setting \( v_- = v_+ = 0 \text{ V} \) and solving for \( v_o \) yields the answer obtained earlier.

b) The conditions match the superposition case II described in (a) but with \( v_{s2} = 1 \text{ V} \).

\[
v_o = 1V = v_{s2} \frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_5}{R_1 || R_2} \right) = 1V \frac{3k}{2k + 3k} \left( 1 + \frac{R_5}{2k \Omega || 3k \Omega} \right)
\]

or

\[
1V = 1V \frac{3}{5} \left( 1 + \frac{R_5}{1.2k \Omega} \right)
\]

or

\[
R_5 = 1.2k \Omega \left( \frac{5}{3} - 1 \right) = 1.2k \Omega \cdot \frac{2}{3} = 0.8k \Omega
\]

c) We make the suggested substitutions for \( v_{s1} \) and \( v_{s2} \):

\[
v_o = \left( v_{cm} - \frac{v_{dm}}{2} \right) \frac{R_5}{R_1} + \left( v_{cm} + \frac{v_{dm}}{2} \right) \frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_5}{R_1 || R_2} \right)
\]

or

\[
v_o = v_{cm} \left[ -\frac{R_5}{R_1} + \frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_5}{R_1 || R_2} \right) \right] + \frac{v_{dm}}{2} \left[ \frac{R_5}{R_1} + \frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_5}{R_1 || R_2} \right) \right]
\]

d)

\[
R_{in} = \frac{v_{s2}}{v_{s2}} = \frac{R_3 + R_4}{R_3 + R_4} = 2k \Omega + 3k \Omega = 5k \Omega
\]