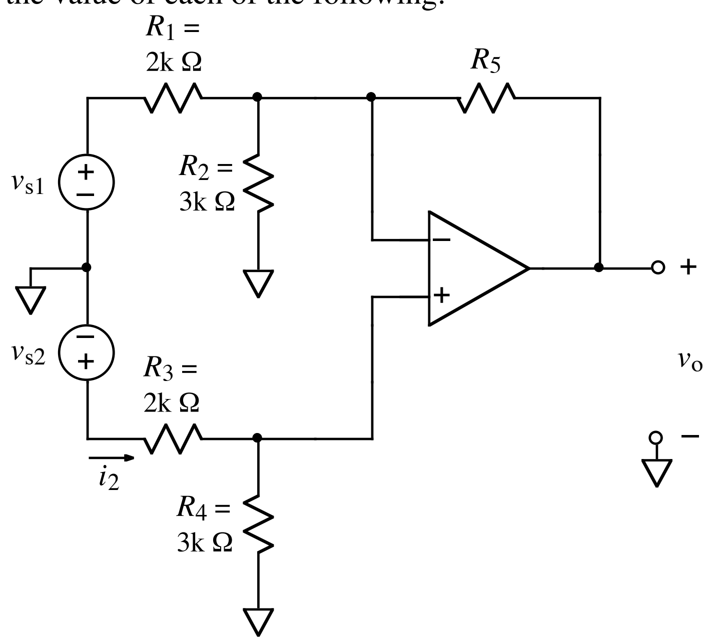
Ex: Find the value of each of the following:



a) The above circuit operates in linear mode. Derive a symbolic expression for v_o. The expression must contain not more than the parameters v_{s1}, v_{s2}, R_1, R_2, R_3, R_4, and R_5.

b) If $v_{s1} = 0$ V and $v_{s2} = 1$ V, find the value of R_5 that will yield an output voltage of $v_o = 1$ V.

c) Derive a symbolic expression for v_o in terms of common mode and differential input voltages:

$$v_{cm} \equiv \frac{(v_{s2} + v_{s1})}{2} \qquad \text{and} \qquad v_{dm} \equiv v_{s2} - v_{s1}$$

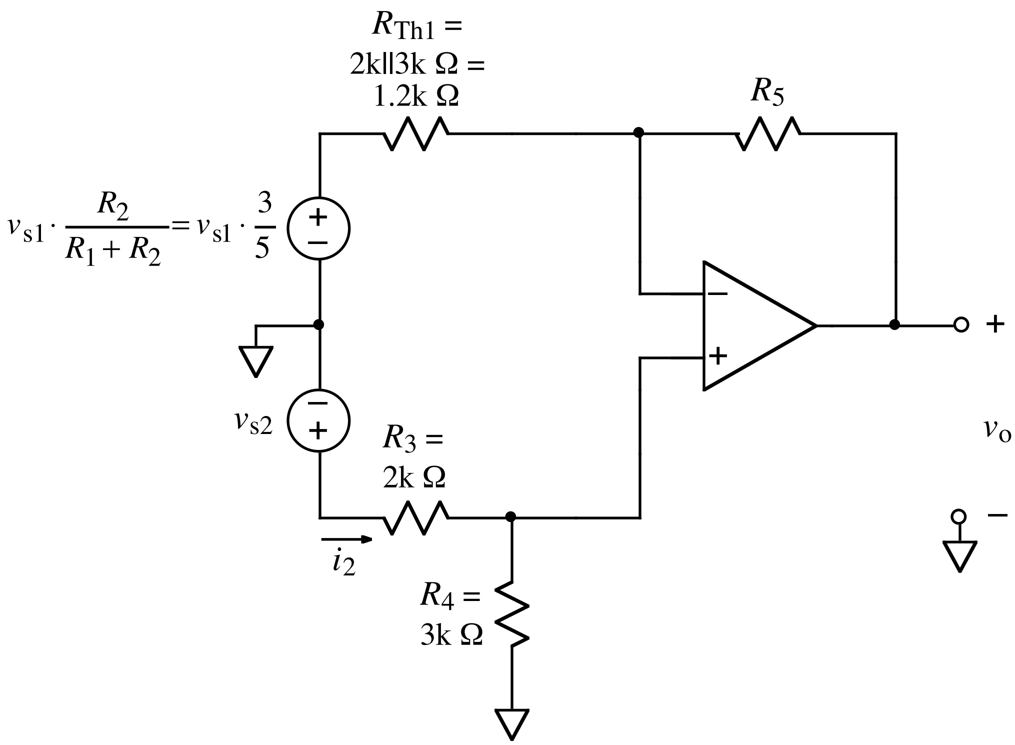
The expression must contain not more than the parameters v_{cm}, v_{dm}, R_1, R_2, R_3, R_4, and R_5. Write the expression as v_{cm} times a term plus v_{dm} times a term. Hint: start by writing v_{s1} and v_{s2} in terms of v_{cm} and v_{dm}:

$$v_{s1} = v_{cm} - \frac{v_{dm}}{2} \qquad \text{and} \qquad v_{s2} = v_{cm} + \frac{v_{dm}}{2}$$

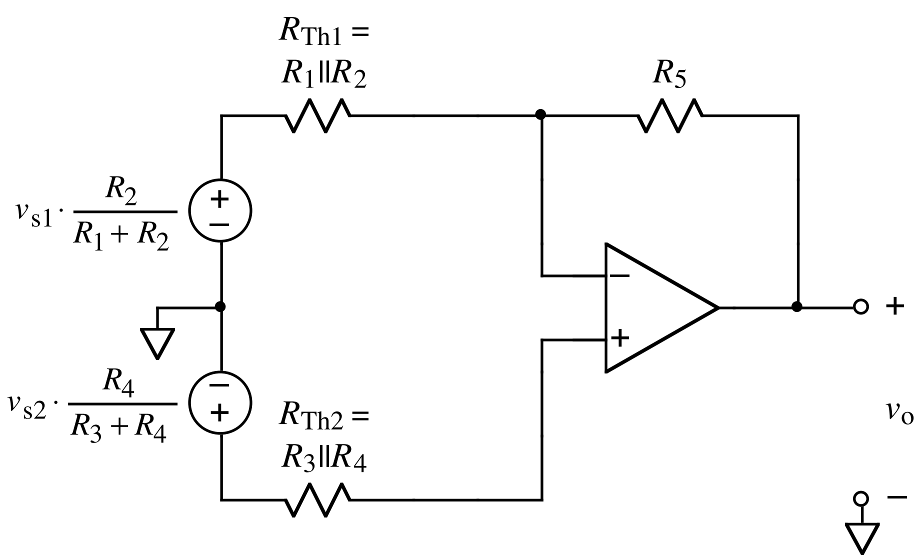
d) Find the numerical value of the circuit's input resistance, R_{in}, as seen by source v_{s2}. In other words, write a formula for voltage, v_{s2}, divided by i_2:

$$R_{in} \equiv \frac{v_{s2}}{i_2}$$

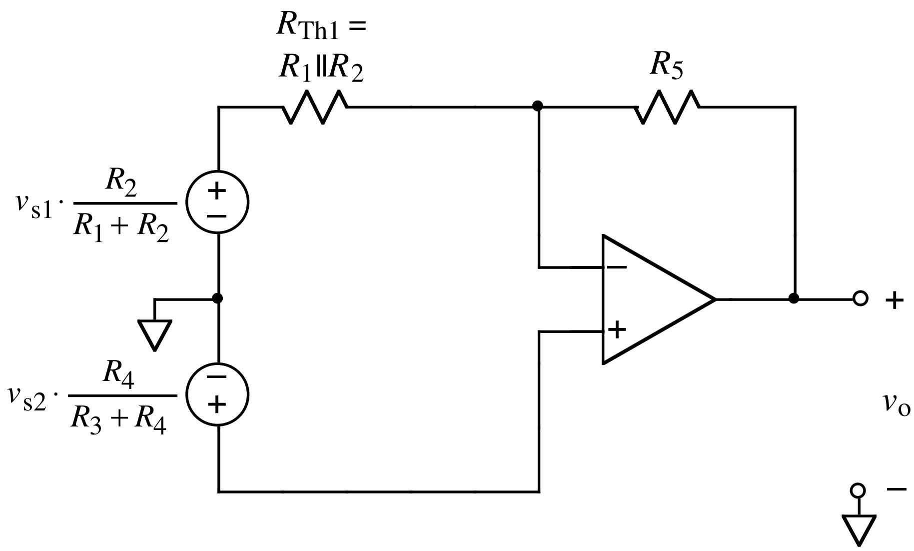
SOL'N: a) Converting the circuitry comprised of v_{s1}, R_1, and R_2 into a Thevenin equivalent yields a standard differential amplifier.



Likewise, converting the circuitry comprising v_{s2}, R_3, and R_4 into a Thevenin equivalent yields further simplification:

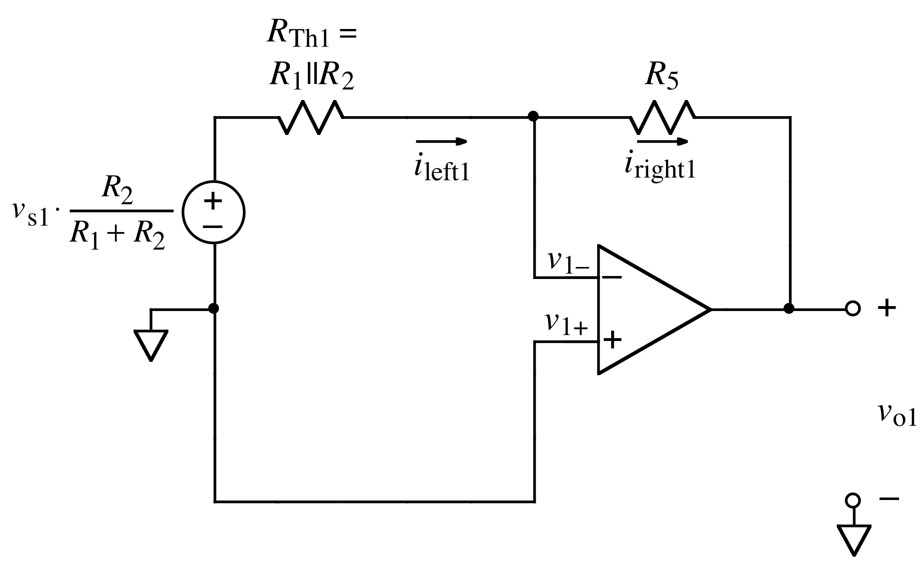


Since no current flows into the + input of the op-amp, R_{Th2} has no voltage drop and may be removed.



Now, applying superposition reveals that this circuit is a combination of a standard negative-gain circuit and positive-gain circuit.

case I: (v_{s1} on, v_{s2} off = wire)



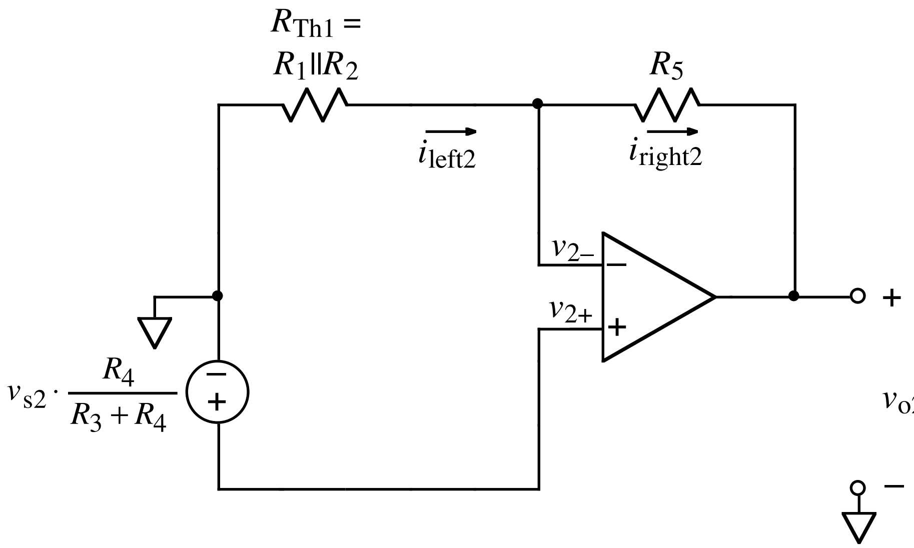
For this circuit, $v_{1-} = v_{1+} = 0$ V. (The "1" stands for "case I".) Using v_{1-} to calculate i_{left1} and i_{right1} gives the following equation:

$$i_{left1} = \frac{v_{s1} \cdot \frac{R_2}{R_1 + R_2}}{R_1 \| R_2} = \frac{-v_{o1}}{R_5} = i_{right1}$$

or

$$v_{o1} = -v_{s1}\frac{R_5}{R_1}$$

case II: (v_{s1} off = wire, v_{s2} on)



For this circuit, $v_{2-} = v_{2+} = v_{s2}\frac{R_4}{R_3 + R_4}$. (The "2" stands for "case II".)

Using v_{2-} to calculate i_{left2} and i_{right2} gives the following equation:

$$i_{left2} = \frac{v_{s2} \cdot \frac{R_4}{R_3 + R_4}}{R_1 \| R_2} = \frac{v_{s2} \cdot \frac{R_4}{R_3 + R_4} - v_{o2}}{R_5} = i_{right2}$$

or

$$v_{o2} = v_{s2}\frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \| R_2}\right)$$

Summing results gives the final result.

$$v_o = -v_{s1}\frac{R_5}{R_1} + v_{s2}\frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \| R_2}\right)$$

Node voltage may also be used. For the v_- node:

$$\frac{v_+}{R_4} + \frac{v_+ - v_{s2}}{R_3} = 0\,\text{A}$$

For the v_+ node:

$$\frac{v_- - v_{s1}}{R_1} + \frac{v_-}{R_2} + \frac{v_- - v_o}{R_5} = 0\,\text{A}$$

Setting $v_- = v_+ = 0$ V and solving for v_o yields the answer obtained earlier.

b) The conditions match the superposition case II described in (a) but with $v_{s2} = 1$ V.

$$v_o = 1\text{V} = v_{s2}\frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \parallel R_2}\right) = 1\text{V}\frac{3\text{k}}{2\text{k} + 3\text{k}}\left(1 + \frac{R_5}{2\text{k}\Omega \parallel 3\text{k}\Omega}\right)$$

or

$$1\text{V} = 1\text{V}\frac{3}{5}\left(1 + \frac{R_5}{1.2\text{k}\,\Omega}\right)$$

or

$$R_5 = 1.2\text{k}\Omega\left(\frac{5}{3} - 1\right) = 1.2\text{k}\Omega \cdot \frac{2}{3} = 0.8\,\text{k}\Omega$$

c) We make the suggested substitutions for v_{s1} and v_{s2}:

$$v_o = -\left(v_{cm} - \frac{v_{dm}}{2}\right)\frac{R_5}{R_1} + \left(v_{cm} + \frac{v_{dm}}{2}\right)\frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \parallel R_2}\right)$$

or

$$v_o = v_{cm}\left[-\frac{R_5}{R_1} + \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \parallel R_2}\right)\right] + \frac{v_{dm}}{2}\left[\frac{R_5}{R_1} + \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_5}{R_1 \parallel R_2}\right)\right]$$

d)

$$R_{in} = \frac{v_{s2}}{\dfrac{v_{s2}}{R_3 + R_4}} = R_3 + R_4 = 2\text{k}\,\Omega + 3\text{k}\,\Omega = 5\text{k}\,\Omega$$