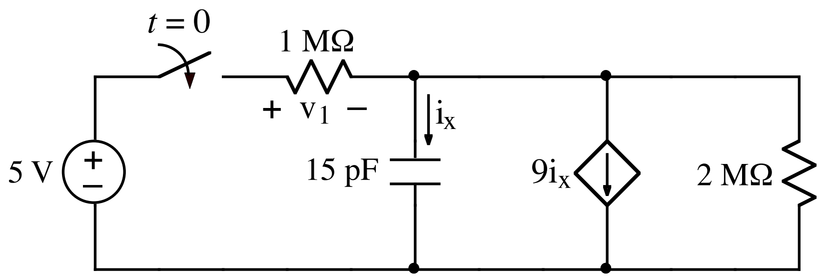
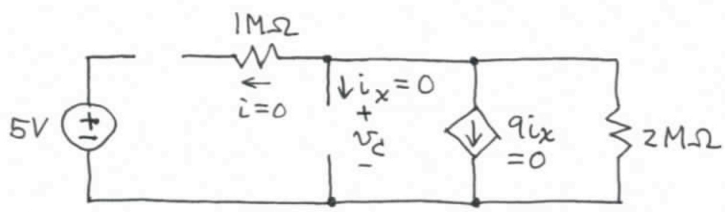
HOMEWORK #9 prob 3 solution

Ex:



After being open for a long time, the switch closes at $t = 0$. Find $v_1(t)$ for $t > 0$.

sol'n: $t = 0^-$ model: (to find $v_c(0^-)$) C = open circuit



The total current flowing out of top node equals zero, and there is no current flowing in the 1 MΩ, the C, and the dependent source. It follows that the current in the 2 MΩ is 0A. By Ohm's law, the voltage drop across the 2 MΩ is $0 \cdot 2\,M\Omega = 0\,V$. This is also the voltage across the C.

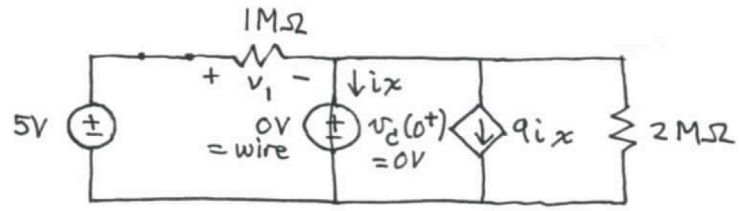
$$\therefore\ v_c(0^-) = 0\,V$$

and

$$v_c(0^+) = v_c(0^-) = 0\,V$$

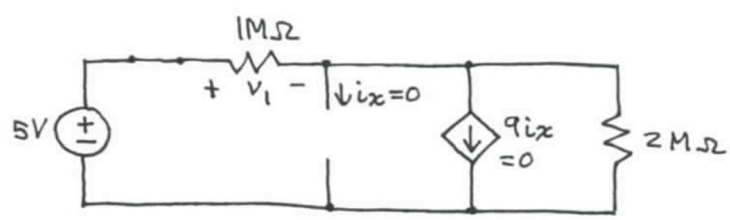
We use this value of $v_c(t = 0^+)$ as a voltage source in the $t = 0^+$ model to find $v_1(0^+)$.

$t=0^+$ model:



From a voltage loop on the left side, we have $v_1(0^+) = 5V$. Note: the components to the right of C are in parallel with the circuitry on the left and directly across the same voltage source, (namely 0V).

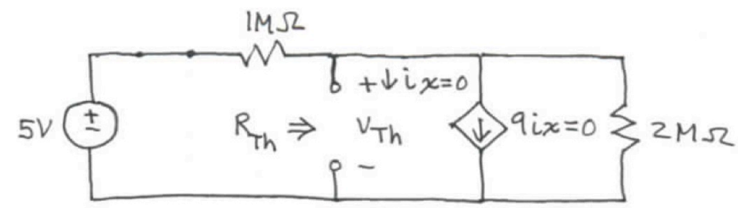
$t \to \infty$ model: (to find $v_1(t\to\infty)$) C= open circ.



The dependent src is off and effectively disappears. This leaves a voltage divider:

$$v_1(t\to\infty) = 5V \cdot \frac{1M\Omega}{1M\Omega + 2M\Omega} = \frac{5}{3}V$$

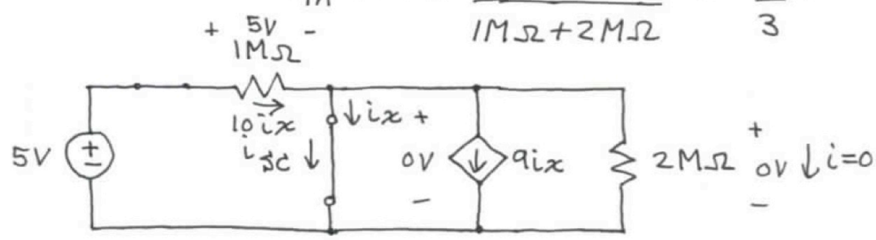
Finally, we have $\tau = R_{Th}C$ where R_{Th} is the Thevenin equivalent resistance seen looking into the terminals where C is connected.



Because there is a dependent source, we find R_{Th} from $R_{Th} = \dfrac{v_{Th}}{i_{sc}}$.

v_{Th}, as always, equals the voltage across the output terminals when nothing is connected across them. Since $i_x = 0$ and $9i_x = 0$, v_{Th} is given by a voltage divider formula:

$$v_{Th} = 5V \cdot \frac{2M\Omega}{1M\Omega + 2M\Omega} = \frac{10}{3}\,V$$



If we short out the output terminals, we have 0V across the 2MΩ resistor. Thus, there is no current in the 2MΩ R.

A current summation for the top node reveals that the current in the 1MΩ must be $10i_x$. From a v-loop on the left side, we also have 5V across the 1MΩ R. Thus, the current in the 1MΩ R is $5V/1M\Omega = 5\,\mu A$. Thus, we have

$$5\,\mu A = 10\,i_x \quad \text{or} \quad i_x = 0.5\,\mu A$$

From the schematic diagram, we see that

$$i_{sc} = i_x = 0.5\ \mu A.$$

$$\therefore\ R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{\frac{10}{3}\ V}{0.5\ \mu A} = \frac{20}{3}\ M\Omega$$

Thus, $\tau = R_{Th} C = \frac{20}{3}\ M\Omega \cdot 15 pF = 100\ \mu s.$

Using the general form of solution, we have

$$v_1(t) = v_1(t\to\infty) + [v_1(t=0^+) - v_1(t\to\infty)]\ e^{-t/\tau}$$

$$v_1(t) = \frac{5}{3}V + \left[5V - \frac{5}{3}V\right] e^{-t/100\mu s}, \quad t>0$$

or $$v_1(t) = \frac{5}{3}V + \frac{10}{3}V\ e^{-t/100\mu s}, \quad t>0$$

Note: A much simpler way to solve this problem is to observe that the $9i_x$ dependent source acts like a capacitor that is 9 times C. Since the C and 9C are in parallel, we have an equivalent capacitance of $10C = 10 \cdot 15 pF = 150 pF$. The dependent source is now gone, and the sol'n is easier to find. The solution, of course is the same as above. $R_{Th} C$ is the same, but $R_{Th} = 1M\Omega \| 2M\Omega$ and $C = 150pF$. $v_1(0^+)$ and $v_1(t\to\infty)$ are the same as before.