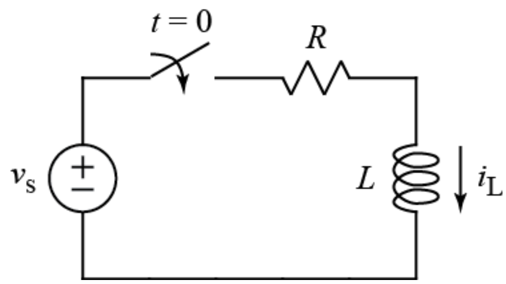
HOMEWORK #8 prob 4 solution

Ex: In the circuit below, the switch closes at $t = 0$ s, $v_s = 2.3$ V, $R = 10$ kΩ, $L = 2$ nH, and $i_L(t = 0) = 5$ A (arising from additional circuitry not shown that is disconnected at time $t = 0$ s).



a) Find an expression for $i_L(t)$ for $t \geq 0$.

b) Find the energy stored in the inductor at time $t = 30$ µs.

SOL'N: a) The following general form of solution applies to any RL circuit with a single inductor:

$$i_L(t \geq 0) = i_L(t \to \infty) + [i_L(t = 0^+) - i_L(t \to \infty)]e^{-t/(L/R_{Th})}$$

The Thevenin resistance, R_{Th}, is for the circuit after $t = 0$ (with the L removed) as seen from the terminals where the L is connected. In the present case, we have $R_{Th} = 10$ kΩ.

$$L/R_{Th} = 2 \text{ nH}/10 \text{ k}\Omega = 0.2 \text{ ps}$$

The value of $i_L(t{=}0)$ is given in the problem as 5 A.

As time approaches infinity, the L current will converge to its final value, and the voltage across the L will cease to change. Thus, $di_L/dt = 0$ and $v_L = 0$, meaning the L will act like a wire. It follows that the current through the L will equal the current through the R, which will equal 2.3 V/10 kΩ = 0.23 mA.

$$i_L(t \to \infty) = 0.23 \text{ mA}$$

Substituting values, we have the following result:

$$i_L(t \geq 0) = 0.23 \text{ mA} + [5\text{ A} - 0.23 \text{ mA}]e^{-t/0.2\text{ps}}$$

b) The energy in an inductor is given by the following formula:

$$w_L = \frac{1}{2} L i_L^2$$

We use the solution to (a) to evaluate $i_L(t)$ at $t = 30$ µs.

$$i_L(t = 30\mu\text{s}) = 0.23 \text{ mA V} - 0.23 \text{ mA} \cdot e^{-30\mu\text{s}/0.2\text{ps}} \approx 0.23 \text{ mA}$$

Using this voltage, we evaluate the energy on the capacitor.

$$w_L = \frac{1}{2} 2\text{nH} \cdot (0.23\text{mA})^2 = 0.053 \text{ fJ} = 53 \text{ aJ}$$