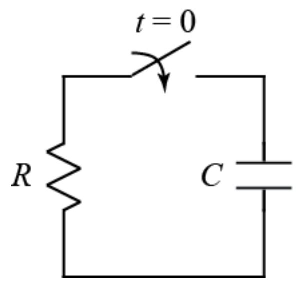
HOMEWORK #8 prob 1 solution

EX: Find the an expression for the voltage, $v_C(t)$, across the capacitor in the circuit below for $t > 0$ if $R = 5\text{ k}\Omega$, $C = 2\ \mu\text{F}$, and $v_C(t = 0) = 8$ V (with + sign of v measurement on top side of C). Note that the switch closes at time $t = 0$.



SOL'N: The same current flows in both the C and R, and the voltages are the same except for a minus sign:

$$i_C = C\frac{dv_C}{dt} = -\frac{v_C}{R}$$

The latter part is a differential equation. The capacitor voltage, v_C, that solves this equation is an exponential. The exponential has the same functional form as its derivative and can satisfy the differential equation at every moment in time.

$$v_C(t) = Ae^{-t/(RC)} = Ae^{-t/(5\text{k}\Omega)(2\,\mu\text{F})} = Ae^{-t/10\,\text{ms}}$$

To satisfy the initial condition as given for $t = 0$, the value of the constant A must be 8 V since the exponential has a value of unity: $e^0 = 1$.

$$v_C(t) = 8e^{-t/10\,\text{ms}}\ \text{V}$$