
EX: a) The following equation describes the current, i_C, through a capacitor as a function of time. Find an expression for the voltage, $v_C(t)$, across the capacitor as a function of time. Assume that $v_C(t = 0) = 2$ V and $C = 1$ μF.

$$i_C(t) = 5e^{-t/8\,\text{ms}} \text{ mA}$$

b) Using your answer to (a), find the time, t, at which v_C is equal to 40 V.

SOL'N: a) We use the defining equation for a capacitor and solve for v in terms of i.

$$i_C = C\frac{dv_C}{dt}$$

First, we multiply both sides by dt.

$$i_C dt = C dv_C$$

Second, we integrate both sides and use limits on the integrals that correspond to the variable of integration for each side and are evaluated at the same points in time for both sides.

$$\int_0^t i_C\, dt = \int_{v_C(t=0)}^{v_C(t)} C\, dv_C$$

The integral on the right side simplifies nicely.

$$\int_0^t i_C\, dt = Cv_C\Big|_{v_C(t=0)}^{v_C(t)} = C\left[v_C(t) - v_C(t=0)\right]$$

or

$$v_C(t) = \frac{1}{C}\int_0^t i_C\, dt + v_C(t=0)$$

The above expression applies to any capacitor in any circuit.

We now substitute the formula given for $i_C(t)$ and the value given for $v_C(t = 0)$ to find $v_C(t)$:

$$v_C(t) = \frac{1}{1\,\mu\text{F}}\int_0^t 5e^{-t/8\,\text{ms}} \text{ mA } dt + 2\text{ V}$$

or

$$v_C(t) = \frac{1}{1\,\mu\text{F}} \frac{1}{\frac{-1}{8\,\text{ms}}} 5e^{-t/8\,\text{ms}}\,\text{mA}\Big|_0^t + 2\,\text{V}$$

or

$$v_C(t) = -\frac{8\,\text{ms}}{1\,\mu\text{F}}\left(5e^{-t/8\,\text{ms}}\,\text{mA-5}\,\text{mA}\right) + 2\,\text{V}$$

or

$$v_C(t) = 40\left(1 - e^{-t/8\,\text{ms}}\right)\text{V} + 2\,\text{V}$$

b) We set the answer to (a) equal to 40 V and solve for t.

$$40\,\text{V} = 40\left(1 - e^{-t/8\,\text{ms}}\right)\text{V} + 2\,\text{V}$$

or

$$38\,\text{V} = 40\left(1 - e^{-t/8\,\text{ms}}\right)\text{V}$$

or

$$\frac{38}{40} = 1 - e^{-t/8\,\text{ms}}$$

We are working towards isolating the exponential term.

$$\frac{38}{40} - 1 = -e^{-t/8\,\text{ms}}$$

We make the exponential positive so we can take the logarithm of both sides.

$$1 - \frac{38}{40} = e^{-t/8\,\text{ms}}$$

or

$$\ln\left(\frac{1}{20}\right) = -t\,/\,8\,\text{ms}$$

or

$$-2.9958 = -t\,/\,8\,\text{ms}$$

or

$$t = 2.9958(8\,\text{ms}) \approx 24\,\text{ms}$$