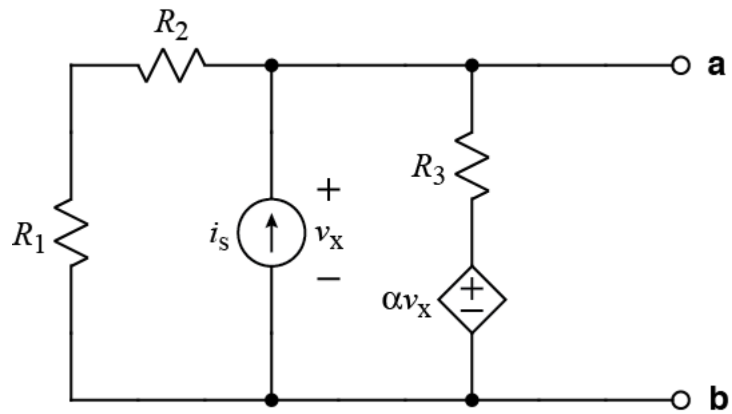
EX:



Find the Thevenin equivalent circuit at terminals a-b.

1) Find the Thevenin equivalent circuit at terminals a-b. v_x must not appear in your solution. The expression must not contain more than circuit parameters α, R_1, R_2, R_3, and i_s. **Note:** $0 < \alpha < 1$.

2) Find the Norton equivalent of the circuit in problem 1.

3) For the circuit in problem 1, assume the following component values:
$i_s = 0.4$ mA, $R_1 = 10$ kΩ, $R_2 = 2$ kΩ, $R_3 = 36$ kΩ, $\alpha=2$

   a) Calculate the value of R_L that would absorb maximum power.

   b) Calculate that value of maximum power R_L could absorb.

SOL'N: 1)

It is always the case that $v_{Th} = v_{a,b}$ no load. We consider first a node-voltage solution. If we place a reference on the bottom rail, then v_{Th} is the voltage for the node consisting of the wire on the top right, which is connected to terminal **a**. We sum the currents flowing out of this node.

$$v_{Th}/(R_1+R_2) + -i_s + (v_{Th} - \alpha v_{Th})/R_3 = 0 \text{ A}$$

Note: $v_x = v_{Th}$ so $\alpha v_x = \alpha v_{Th}$

Now we solve for v_{Th}.

$$v_{Th}\left(\frac{1}{R_1+R_2}+\frac{1-\alpha}{R_3}\right)=i_s$$

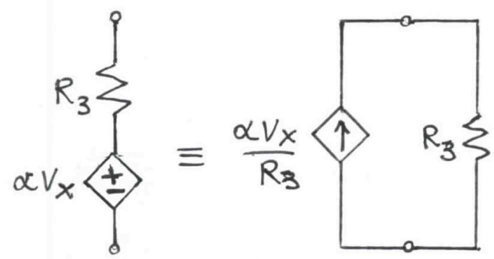
Multiplying both sides by $(R_1+R_2)R_3$ gives

$$v_{Th}\left[R_3+(1-\alpha)(R_1+R_2)\right]=i_s(R_1+R_2)R_3$$

or

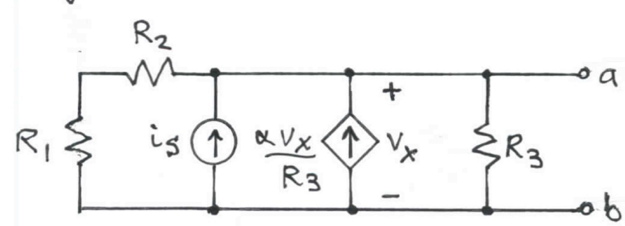
$$v_{Th}=\frac{i_s(R_1+R_2)R_3}{R_3+(1-\alpha)(R_1+R_2)}$$

An alternate approach (suggested by Norm Gifford) is to first replace the dependent source and R_3 with a Norton equivalent.



Thev. equiv.   Norton equiv.

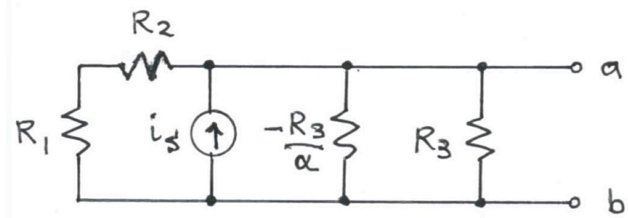
With the Norton equivalent, we have the following circuit:



We have both the voltage and current for the dependent source as functions of v_x, allowing us to find an equivalent R value.

$$R_{eq} = -\frac{V_x}{\frac{\alpha V_x}{R_3}} = -\frac{R_3}{\alpha}$$

Our new circuit model:



We now find $v_{Th} = v_{a,b}$ no load using Ohm's law:

$$v_{Th} = i_s \cdot (R_1 + R_2) \| -\frac{R_3}{\alpha} \| R_3$$

We compare this to the previous answer by simplifying the expression for parallel resistances.

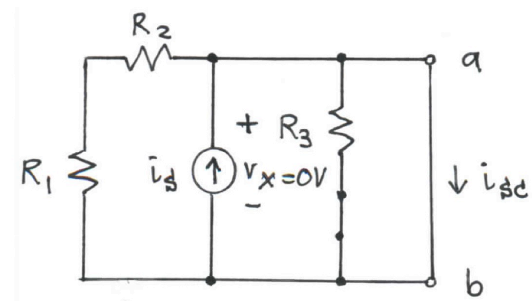
$$(R_1+R_2) \| -\frac{R_3}{\alpha} \| R_3 = \frac{1}{\frac{1}{R_1+R_2} - \frac{\alpha}{R_3} + \frac{1}{R_3}}$$

$$= \frac{(R_1+R_2) R_3}{R_3 + (1-\alpha)(R_1+R_2)}$$

So $v_{Th} = i_s \dfrac{(R_1 + R_2) R_3}{R_3 + (1-\alpha)(R_1 + R_2)}$ as before.

To find R_{Th}, we can short the output from a to b, find the short circuit current, i_{sc}, and then use $R_{Th} = v_{Th} / i_{sc}$.

When we short a to b, we have $v_x = 0\,V$, turning the dependent source into a wire.



Note: $v_x = 0\,V$ since we have a wire from a to b.

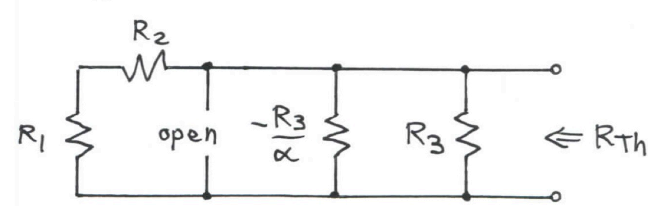
We have a current divider, but all the current will flow in the short circuit (wire) since it has zero resistance.

$$i_{sc} = i_s$$

So $\quad R_{Th} = \dfrac{v_{Th}}{i_{sc}} = \dfrac{v_{Th}}{i_s} = \dfrac{(R_1 + R_2)\,R_3}{R_3 + (1-\alpha)(R_1 + R_2)}$

An alternate approach is to observe that the R_{eq} that replaced the dependent source is valid regardless of what linear circuit we connect across a and b. That is, v_x is still across the Norton equivalent source $-\alpha v_x / R_3$.

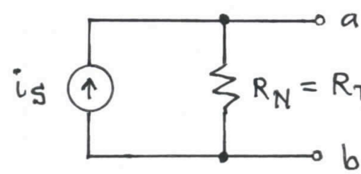
In that case, we can turn off i_s and look in from the a and b terminals to find R_{Th}.



So $\quad R_{Th} = (R_1 + R_2) \,\|\, -\dfrac{R_3}{\alpha} \,\|\, R_3 \quad$ as before.

SOL'N: 2)

It is always the case that $i_N = \dfrac{v_{Th}}{R_{Th}}$ and $R_N = R_{Th}$. We find $i_N = i_s = \dfrac{v_{Th}}{R_{Th}}$ from (a).



$$R_N = R_{Th} = \frac{(R_1 + R_2)\, R_3}{R_3 + (1-\alpha)(R_1 + R_2)}$$

SOL'N: 3.a) The maximum power transfer occurs when the load resistance equals the Thevenin equivalent resistance, which may be shown by writing an expression for the power as follows and setting the derivative with respect to R_L equal to zero:

$$p = iv = \frac{v_{Th}}{R_{Th} + R_L} \cdot \frac{v_{Th} R_L}{R_{Th} + R_L}$$

$$\frac{dp}{dR_L} = 0 \;\Rightarrow\; R_L = R_{Th}$$

From the answers to (1) and (2) above, we calculate the R_{Th} for the values given.

$$R_{Th} = \frac{(R_1 + R_2) R_3}{R_3 + (1-\alpha)(R_1 + R_2)} = \frac{(10\,\text{k}\Omega + 2\,\text{k}\Omega)36\,\text{k}\Omega}{36\,\text{k}\Omega + (1-2)(10\,\text{k}\Omega + 2\,\text{k}\Omega)}$$

or

$$R_{Th} = \frac{(12\,\text{k}\Omega)36\,\text{k}\Omega}{36\,\text{k}\Omega - 12\,\text{k}\Omega} = \frac{12\,\text{k}\Omega}{12\,\text{k}\Omega} \cdot \frac{36\,\text{k}\Omega}{2} = 18\,\text{k}\Omega$$

Thus,

$$R_L = R_{Th} = 18\,\text{k}\Omega$$

b) When $R_{\mathrm{Th}} = R_{\mathrm{L}}$, the formula for power always simplifies as follows:

$$p_{\max} = \frac{v_{\mathrm{Th}}^2 R_{\mathrm{Th}}}{\left(R_{\mathrm{Th}} + R_{\mathrm{Th}}\right)^2} = \frac{v_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}}$$

Using the v_{Th} formula from (1) with values given, we get the following value:

$$v_{\mathrm{Th}} = \frac{i_{\mathrm{s}}\left(R_1 + R_2\right)R_3}{R_3 + (1-\alpha)\left(R_1 + R_2\right)} = \frac{0.4\,\mathrm{mA}\left(10\,\mathrm{k\Omega} + 2\,\mathrm{k\Omega}\right)36\,\mathrm{k\Omega}}{36\,\mathrm{k\Omega} + (1-2)\left(10\,\mathrm{k\Omega} + 2\,\mathrm{k\Omega}\right)}$$

or

$$v_{\mathrm{Th}} = i_{\mathrm{s}} R_{\mathrm{Th}} = 0.4\,\mathrm{mA}(18\,\mathrm{k\Omega}) = 7.2\,\mathrm{V}$$

Thus,

$$p_{\max} = \frac{v_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}} = \frac{(7.2\,\mathrm{V})^2}{4(18\,\mathrm{k\Omega})} = 0.72\,\mathrm{mW}\,.$$